Formulations of a carsharing pricing and relocation problem

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In this lecture

- Carsharing and the fleet imbalance problem
- A mathematical model for carsharing joint pricing & relocation activities
- A reformulation
- Some results
- GAMS implementation

Carsharing

Carsharing:

- A company (CSO) owns a large fleet of vehicles
- Makes it available for short term rentals
- Users find and rent cars using an app
- Users pay based on time/distance (plus possibly zone prices)

Carsharing

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- transport costs (no more bollo, insurance and tagliando!)
- affordable mobility for disadvantaged groups
- Growing in popularity, and likely to grow further with EVs.

Carsharing

Current configuration:

- users demand high flexibility
- carsharing is commonly designed for *on-demand*, *short-term*, *one-way* usage

Carsharing

This means troubles for the carsharing operator (CSO)!

On-demand: the CSO unaware of when, where and for how long new rentals will occur.

One-way: frequent imbalances in the distribution of vehicles, vehicles are not where you want them to be.

A central task for a CSO is to provide a distribution of vehicles in the business area compatible with demand tides and oscillations.

Carsharing

As a prime form of response CSOs initiate staff-based vehicle relocations before shortages occur.

That is, CSO's staff reach designated cars and drives them to different places.

This alone is inherently costly and inefficient!

Carsharing

The novel idea is to manipulate demand throught prices.

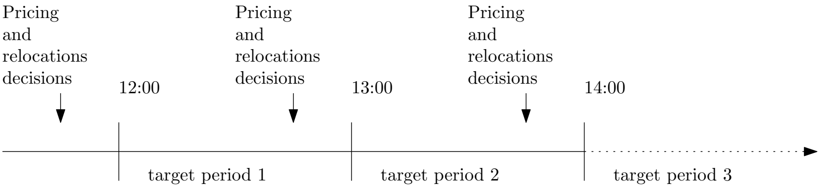
Users choose among different transport modes (e.g., metro, carsharing, bike, bus) that vary in a number of key attributes including price.

Carsharing

We provide a model for the problem of simultaneously setting carsharing prices and deciding relocations.

Assumptions

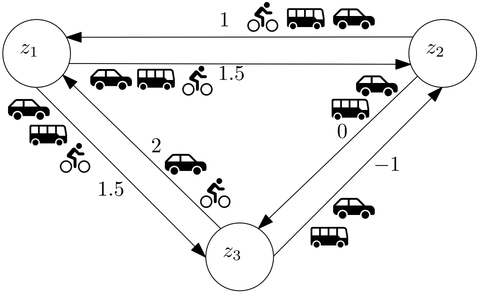
Target periods



Assumptions

Zones

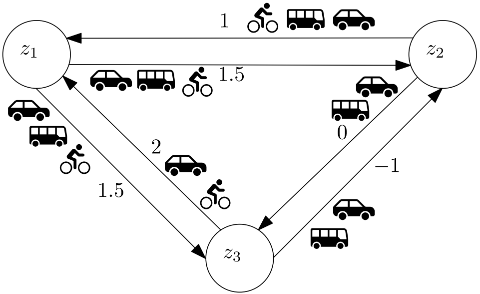
per-minute fee 0.20 drop-off fees = {-1,0,1,1.5,2}



Assumptions

Drop-off fee + per-minute fee

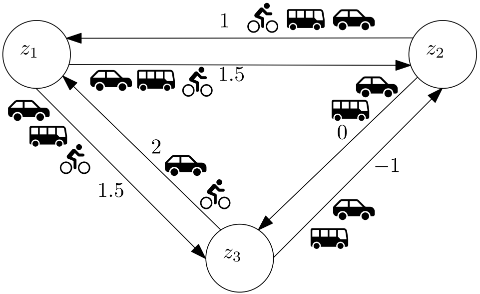
per-minute fee 0.20 drop-off fees = {-1,0,1,1.5,2}



Assumptions

Alternative transport services

per-minute fee 0.20 drop-off fees = {-1,0,1,1.5,2}



The problem

Given

- a target period
- a fleet of shared cars and their current position
- the cumulative mobility demand between each pair of zones in the target period,
- usage and relocation costs
- a model of customers preferences

decide

- the drop-off fees to apply during the target period
- the relocations to perform

Basic elements

The urban area is represented by a set $\mathcal{I}$ of zones.

The CSO offers a set of shared vehicles $\mathcal{V}$.

There is a finite set $\mathcal{L}$ of drop-off fees the CSO is considering.

The city counts a set $\mathcal{A}$ of alternative transport services.

$\mathcal{K}$ is the set of customers ($\mathcal{K}_i \subseteq \mathcal{K}$ and $\mathcal{K}_{ij} \subseteq \mathcal{K}_i$).

Key decisions

z_{vi} is equal to 1 if vehicle v is made available for rental in (possibly relocated to) zone i in the target period, 0 otherwise.

$$\sum_{i \in \mathcal{I}} z_{vis} = 1 \qquad v \in \mathcal{V}$$

Key decisions

λ_{ijl} is equal to 1 if fee l is applied between zone i and and zone j, 0 otherwise.

$$\sum_{l \in \mathcal{L}} \lambda_{ijl} = 1 \qquad i \in \mathcal{I}, j \in \mathcal{J}$$

Key decisions

Let decision variable p_{vij} be the price of service $v \in \mathcal{V} \cup \mathcal{A}$ between zones i and j.

For carsharing

$$p_{vij} = P_v^V T_{vij}^{CS} + \sum_{l \in \mathcal{L}} L_l \lambda_{ijl} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I}$$

For alternative services

$$p_{vij} = P_{vij} \qquad \forall v \in \mathcal{A}, i, j \in \mathcal{I}$$

Customers response

Each customer has unique preferences.

Choses the transport service that provides them the highest utility.

This utility is known to the customer but not to the CSO.

Customers response

The CSO is not aware of the utility provided by the different services to each customer.

The CSO is aware of a number of characteristics of the services (e.g., price, travel time, waiting time).

The CSO can model utility as

$$F_k(p_{vij}, \pi^1_{vij}, \ldots, \pi^N_{vij}) + \tilde{\xi}_{kv}$$

Constraints – Utility

Let u_{ijkv} be the utility obtained by customer $k \in \mathcal{K}$ when moving from i to $j \in \mathcal{I}$ using service $v \in \mathcal{V} \cup \mathcal{A}$.

$$u_{ijkv} = F_k(p_{vij}, \pi^1_{vij}, \ldots, \pi^N_{vij}) + \xi_{kv} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v \in \mathcal{V} \cup \mathcal{A}$$

ξ_{kv} is a realization of $\tilde{\xi}_{kv}$.

Constraints – Choices

Decision variable w_{ijkv} is equal to 1 if customer k chooses service v, 0 otherwise.

$$\sum_{v \in \mathcal{V} \cup \mathcal{A}} w_{ijkv} = 1 \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}$$

Constraints – Availability

Availablility.

y_{ikv} is 1 if $v \in \mathcal{V} \cup \mathcal{A}$ is available to customer $k \in \mathcal{K}_i$, 0 otherwise.

Alternative services $v \in \mathcal{A}$ are available to all k if available at all

$$y_{ikv} = Y_{vi} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{A}$$

For carsharing

$$y_{ikv} \leq z_{iv} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{V}$$

Constraints – Availability

The first who arrives get the car

$$y_{ikv} \leq y_{i(k-1)v} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{V}$$

A vehicle becomes unavailable for a customer if any customer has arrived before

$$z_{iv} - y_{ikv} = \sum_{j \in \mathcal{I}} \sum_{q \in \mathcal{K}_{ij} : q < k} w_{ijqv} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{V}$$

Constraints – Choices

Can chose a service if available

$$w_{ijkv} \leq y_{ikv} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v \in \mathcal{V} \cup \mathcal{A}$$

Constraints – Choices

Customer choose the service which the highest utility

$$w_{ijkv} \leq \mu_{ijvwk} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A}$$

$$M_{ijk}\nu_{ivwk} - 2M_{ijk} \leq u_{ijkv} - u_{ijkw} - M_{ijk}\mu_{ijvwk}$$
$$\forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A}$$

and

$$u_{ijkv} - u_{ijkw} - M_{ijk}\mu_{ijvwk} \leq (1 - \nu_{ivwk})M_{ijk}$$
$$\forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A}$$

Constraints – Choices

$$\mu_{ijvwk} + \mu_{ijwvk} \leq 1 \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A}$$

A service can be preferred only if offered

$$\mu_{ijvwk} \leq y_{ikv} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A}$$

Constraints – Choices

α_{ijkvl} be equal to 1 if fare l is applied between i and j and customer k chooses shared car v, 0 otherwise.

Relationship between λ_{ijl} and w_{ijkv} and α_{ijkvl}

$$\lambda_{ijl} + w_{ijkv} \leq 1 + \alpha_{ijkvl} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, l \in \mathcal{L}$$
$$\alpha_{ijkvl} \leq \lambda_{ijl} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, l \in \mathcal{L}$$
$$\alpha_{ijkvl} \leq w_{ijkv} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, l \in \mathcal{L}$$

That is, α_{ijkvl} is forced to take value 1 as soon as both λ_{ijl} and w_{ijkv} take value one, and value 0 as soon as either λ_{ijl} or w_{ijkv} take value 0.

Objective function

$$\max \quad -\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C^R_{vi} z_{vi}$$

$$\sum_{v\in\mathcal{V}}\sum_{(i,j)\in\mathcal{I}\times\mathcal{I}} \left(P^V T^{CS}_{ij} - C^U_{ij}\right) \sum_{k\in\mathcal{K}_{ij}} w_{ijkv}$$

$$+\sum_{v\in\mathcal{V}}\sum_{(i,j)\in\mathcal{I}\times\mathcal{I}}\sum_{k\in\mathcal{K}_{ij}}\sum_{l\in\mathcal{L}} L_{ijl}\alpha_{ijkvl}$$

Summarizing

Maximize rental profits such that

- Each car is relocate at most once
- Exactly one drop-off fee is choses between each O-D pair
- Customers choose the service yielding the highest utility (only one)
- A service is chosen if available
- The first customer gets the car

A reformulation

From customers to requests.

A request is a customer who will choose CS if the price is low enoguh.

More precisely, a customer for which there exists a price level at which they would choose carsharing.

Set $\mathcal{R}$ of requests, parameters $i(r)$, $j(r)$, $k(r)$, and $l(r)$.

Reformulation – Requests

Generating requests:

- For each $k \in \mathcal{K}$
- For each level $l \in \mathcal{L}$
- If CS utility at level $l >$ utility alternative services
- Create a request r and add it to $\mathcal{R}$

A reformulation

Decision variables:

- y_{vrl} equal to 1 if request r is satisfied by vehicle v at level l, 0 otherwise.
- z_{vi} if vehicle v is made available at zone i, 0 otherwise.
- λ_{ijl} be equal to 1 if drop-off level l is applied between i and j, 0 otherwise.

A reformulation

$$\max \sum_{r\in\mathcal{R}} \sum_{v\in\mathcal{V}} \sum_{l\in\mathcal{L}_r} R_{vrl} y_{vrl} - \sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{I}} C^R_{vi} z_{vi} \tag{1}$$

$$\sum_{v\in\mathcal{V}} \sum_{l\in\mathcal{L}_r} y_{vrl} \leq 1 \qquad r \in \mathcal{R} \tag{2}$$

$$\sum_{r\in\mathcal{R}} \sum_{l\in\mathcal{L}_r} y_{vrl} \leq 1 \qquad v \in \mathcal{V} \tag{3}$$

$$\sum_{i\in\mathcal{I}} z_{vi} = 1 \qquad v \in \mathcal{V} \tag{4}$$

$$\sum_{l\in\mathcal{L}_{r_1}} y_{v,r_1,l} - z_{v,i(r_1)} + \sum_{r_2\in\mathcal{R}_{r_1}} \sum_{l\in\mathcal{L}_{r_2}} y_{v,r_2,l} \leq 0 \qquad r_1 \in \mathcal{R}, v \in \mathcal{V} \tag{5}$$

$$y_{v,r_1,l_1} \geq \lambda_{i(r_1),j(r_j),l_1} + z_{v,i(r_1)} - \sum_{r_2\in\mathcal{R}_{r_1}} \sum_{l_2\in\mathcal{L}_{r_2}} y_{v,r_2,l_2} - \sum_{v_1\in\mathcal{V}:v_1\neq v} y_{v_1,r_1,l_1} - 1 \qquad r_1 \in \mathcal{R}, v \in \mathcal{V}, l_1 \in \mathcal{L}_{r_1} \tag{6}$$

$$\sum_{l\in\mathcal{L}} \lambda_{ijl} = 1 \qquad i \in \mathcal{I}, j \in \mathcal{J} \tag{7}$$

$$\sum_{v\in\mathcal{V}} y_{vrl} \leq \lambda_{i(r),j(r),l} \qquad r \in \mathcal{R}, l \in \mathcal{L}_r \tag{8}$$

$$y_{vrl} \in \{0,1\} \qquad r \in \mathcal{R}, v \in \mathcal{V}, l \in \mathcal{L}_r \tag{9}$$

$$z_{vi} \in \{0,1\} \qquad i \in \mathcal{I}, v \in \mathcal{V} \tag{10}$$

$$\lambda_{ijl} \in \{0,1\} \qquad i \in \mathcal{I}, j \in \mathcal{I}, l \in \mathcal{L}. \tag{11}$$

Comparing formulations

Instances: a case studies that replicate the carsharing system in the city of Milan (soon available online).

20 to 100 shared vehicles, 50 to 300 customers. Alternative services: Public transport and Bicycles.

Comparing formulations

For each $k \in \mathcal{K}$ traveling between i and j with transport service v, the utility is

$$F_k(p_{vij}, T^{CS}_{vij}, T^{PT}_{vij}, T^{B}_{vij}, T^{Walk}_{vkij}, T^{Wait}_{vij}) = \beta^P_k p_{vij} + \beta^{CS}_k T^{CS}_{vij} + \beta^{PT}_k T^{PT}_{vij} + \tau(T^B_{vij})\beta^B_k T^B_{vij} + \tau(T^{Walk}_{vij})\beta^{Walk}_k T^{Walk}_{vij} + \beta^{Wait}_k T^{Wait}_{vij}$$

Comparing formulations

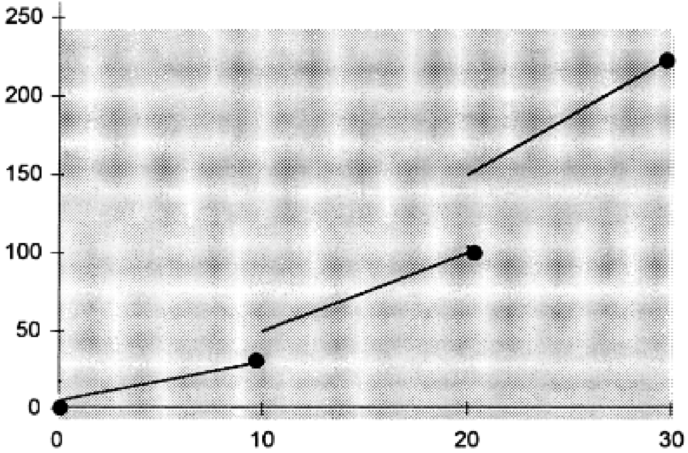


Figure: Piecewise disutility of walking

Comparing formulations

Table: Average solve time [sec] and percentage of problems solved for the small instances. The symbol "–" indicates that the solution process failed for excessive consumption of memory resources.

		Time [sec]		Solved [%]	
V	K	F1	F2	F1	F2
20	50	67.485	0.465	100	100
20	75	187.209	0.506	80	100
20	100	309.379	0.658	40	100
35	50	342.699	0.784	20	100
35	75	360.953	1.230	0	100
35	100	362.098	1.771	0	100
50	50	361.400	1.432	0	100
50	75	361.060	2.048	0	100
50	100	363.038	2.635	0	100
50	200	–	6.763	–	100
50	300	–	13.680	–	100
75	200	368.963	10.964	0	100
75	300	397.119	18.955	0	100
100	200	384.174	19.546	0	100
100	300	376.794	34.848	0	100

Comparing formulations

Table: Optimal objective value compared to the optimal objective value of the LP relaxations for the instances with $V = 20$ and $K = 50$.

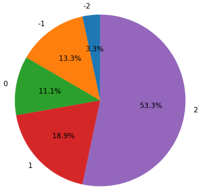
Instance	V	K	Optimal objective value	LP objective value F1	LP objective value F2
1	20	50	53.58	124.22	53.58
2	20	50	40.64	128.12	40.64
3	20	50	25.94	117.19	25.94
4	20	50	25.94	119.35	25.94
5	20	50	38.44	124.75	38.44

Solutions

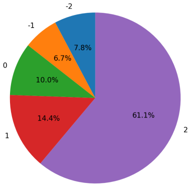
Table: Comparison of the solutions with and without dynamic pricing on the instances with 50 vehicles and 600 customers.

Distribution	Metric	With dynamic pricing	Without dynamic pricing
D1	Expected Profit [%]	100	81.78
	% of vehicles Relocated	26.0	10.0
	Min $\|\mathcal{R}\|$	167	80
	Max $\|\mathcal{R}\|$	195	107
	Expected % Requests satisfied	24	42
D2	Expected Profit [%]	100	66.06
	% of vehicles Relocated	22.0	2.0
	Min $\|\mathcal{R}\|$	168	81
	Max $\|\mathcal{R}\|$	187	105
	Expected % Requests satisfied	26	49
D3	Expected Profit [%]	100	65.05
	% of vehicles Relocated	18.0	6.0
	Min $\|\mathcal{R}\|$	167	80
	Max $\|\mathcal{R}\|$	195	107
	Expected % Requests satisfied	26	49
D4	Expected Profit [%]	100	66.36
	% of vehicles Relocated	10.0	0.0
	Min $\|\mathcal{R}\|$	168	81
	Max $\|\mathcal{R}\|$	187	105
	Expected % Requests satisfied	26	48

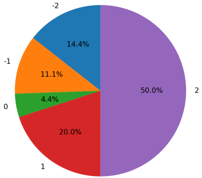
Solutions



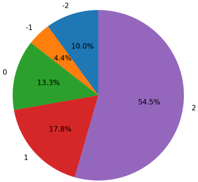
(a) Distribution $D1$



(b) Distribution $D2$



(c) Distribution $D3$



(d) Distribution $D4$

Take-aways

- Carsharing one of the current trends in transportation
- New challenges to face –> more need for optimization!
- Same problem – different models
- The modeling choice can make the difference
- What we do has a significant impact on the company's performance