Benders decomposition for a carsharing pricing problem under uncertainty

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In a nutshell

- A possible way to dynamically adjust prices in carsharing services
- A stochastic programming model
- An exact algorithm
- Some results

Focus:

- One-way car-sharing service

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Problem:

- Imbalanced distributions

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A possible fix:

- Adjust prices as the distribution changes

Take into account

- Users choose among different transport modes (e.g., metro, carsharing, bike, bus)

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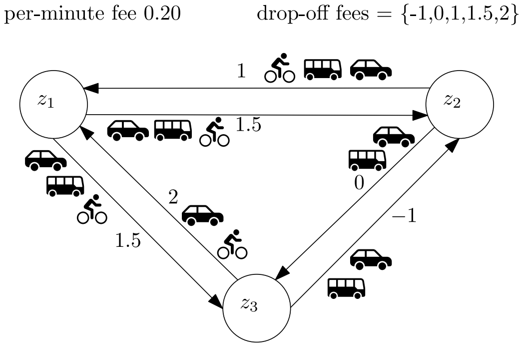
- Users have preferences (price, walking distance, crowd, time, weather...)
- Preferences are not fully known to the CS company

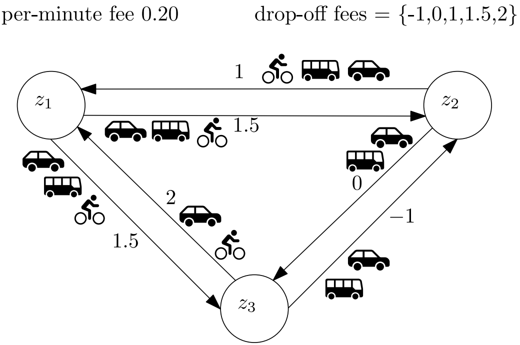
Assumption

Zones



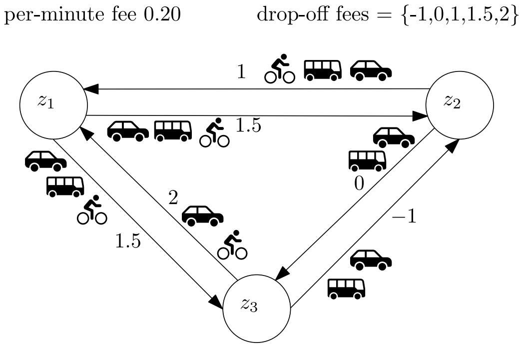
Assumption

Drop-off fee + per-minute fee



Assumption

Alternative transport services



The problem

Given

- A target period (e.g., 14:00 - 15:00)

Decide

Maximize profits

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- Cumulative mobility demand between each pair of zones in the target period

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- A target period (e.g., 14:00 - 15:00)
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- Usage and relocation costs

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- A target period (e.g., 14:00 - 15:00)
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- A model of customers preferences

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Decide

- the drop-off fees for the target period (λ_{ijl})

Maximize profits

The problem

Given

- A target period (e.g., 14:00 - 15:00)
- Current position of the vehicles
- Cumulative mobility demand between each pair of zones in the target period
- Usage and relocation costs
- A model of customers preferences

Decide

- the drop-off fees for the target period (λ_{ijl})
- the relocations to perform (z_{vi})

Maximize profits

The model

$$\max - \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{I}} C^R_{vi} z_{vi} + Q(z, \lambda)$$

$$\sum_{i \in \mathcal{I}} z_{vi} = 1 \qquad v \in \mathcal{V}$$

$$\sum_{l \in \mathcal{L}} \lambda_{ijl} = 1 \qquad i, j \in \mathcal{I}$$

$$z_{vi} \in \{0, 1\} \qquad i \in \mathcal{I}, v \in \mathcal{V}$$

$$\lambda_{ijl} \in \{0, 1\} \qquad i, j \in \mathcal{I}, l \in \mathcal{L}.$$

The model

$$Q(z, \lambda) := \mathbb{E}_{\tilde{\xi}}\Big[Q(z, \lambda, \xi)\Big]$$

$\tilde{\xi}$ models the uncertain preferences of the customers

Customers response

Each customer has unique preferences

$$F_{kv}(p_v, \pi^1_{vij}, \ldots, \pi^N_{vij}) + \tilde{\xi}_{kv}$$

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$$\tilde{\xi} = (\xi_{kv})_{k \in \mathcal{K}, v \in \mathcal{V}}$$

Customers response

Given a realization ξ of $\tilde{\xi}$ we know what customers prefer.

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Given a realization ξ of $\tilde{\xi}$ we know what customers prefer.

For each ξ realization (scenario) we can spot the set of customers $\mathcal{R}(\xi)$ which would use CS at some price level.

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- First-come-first-served

Second stage

$$Q(z,\lambda,\xi) = \max \sum_{r\in\mathcal{R}(\xi)} \sum_{v\in\mathcal{V}} \sum_{l\in\mathcal{L}_r(\xi)} R_{vrl} y_{vrl}$$

$$\sum_{v\in\mathcal{V}} \sum_{l\in\mathcal{L}_r(\xi)} y_{vrl} \leq 1 \qquad r \in \mathcal{R}(\xi)$$

$$\sum_{r\in\mathcal{R}(\xi)} \sum_{l\in\mathcal{L}_r(\xi)} y_{vrl} \leq 1 \qquad v \in \mathcal{V}$$

$$\sum_{l\in\mathcal{L}_{r_1}(\xi)} y_{v,r_1,l} + \sum_{r_2\in\mathcal{R}_{r_1}(\xi)} \sum_{l\in\mathcal{L}_{r_2}(\xi)} y_{v,r_2,l} \leq z_{v,i(r_1)} \qquad r_1 \in \mathcal{R}(\xi), v \in \mathcal{V}$$

$$y_{v,r_1,l_1} + \sum_{r_2\in\mathcal{R}_{r_1}(\xi)} \sum_{l_2\in\mathcal{L}_{r_2}(\xi)} y_{v,r_2,l_2} + \sum_{v_1\in\mathcal{V}:v_1\neq v} y_{v_1,r_1,l_1}$$

$$\geq \lambda_{i(r_1),j(r_j),l_1} + z_{v,i(r_1)} - 1 \qquad r_1 \in \mathcal{R}(\xi), v \in \mathcal{V}, l_1 \in \mathcal{L}_{r_1}(\xi)$$

$$\sum_{v\in\mathcal{V}} y_{vrl} \leq \lambda_{i(r),j(r),l} \qquad r \in \mathcal{R}(\xi), l \in \mathcal{L}_r(\xi)$$

$$y_{vrl} \in \{0,1\} \qquad r \in \mathcal{R}(\xi), v \in \mathcal{V}, l \in \mathcal{L}_r(\xi)$$

Benders decomposition

Two-Stage Stochastic Integer Program, with integers at both stages.

Benders decomposition's key ingredients:

- Fast exact algorithm for the second-stage sub-problems ($\mathcal{O}\big(|\mathcal{R}(\xi_s)| \times |\mathcal{V}|\big)$)

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- Complete recourse
- Ad-hoc optimality cuts
- Multi-cut
- Something to help optimality cuts

Benders decomposition

Master problem

$$\max -\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{I}} C^R_{vi} z_{vi} + \sum_{s\in\mathcal{S}} \pi_s \phi_s$$

$$\sum_{i\in\mathcal{I}} z_{vis} = 1 \qquad v \in \mathcal{V}$$

$$\sum_{l\in\mathcal{L}} \lambda_{ijl} = 1 \qquad i \in \mathcal{I}, j \in \mathcal{J}$$

$$z_{vi} \in \{0,1\} \qquad i \in \mathcal{I}, v \in \mathcal{V}$$

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$$\phi_s \text{ free} \qquad s \in \mathcal{S}.$$

Benders decomposition

Proposition

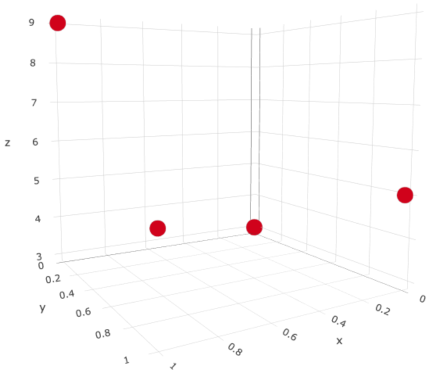
Let (z^t, λ^t) be the t-th feasible solution to MP, and $Q(z, \lambda, \xi_s)$ its second-stage value for scenario s. The set of cuts

$$\phi_s \leq \Big(Q(z, \lambda, \xi_s) - U_s \Big) \Big(\sum_{(v,i) \in \mathcal{Z}_t^+} z_{vi} - \sum_{(v,i) \in \mathcal{Z}_t^-} z_{vi} + \sum_{(i,j,l) \in \Lambda_t^+} \lambda_{ijl} - \sum_{(i,j,l) \in \Lambda_t^-} \lambda_{ijl} \Big) + U_s - \Big(Q(z, \lambda, \xi_s) - U_s \Big) \Big(|\mathcal{Z}_t^+| + |\Lambda_t^+| - 1 \Big)$$

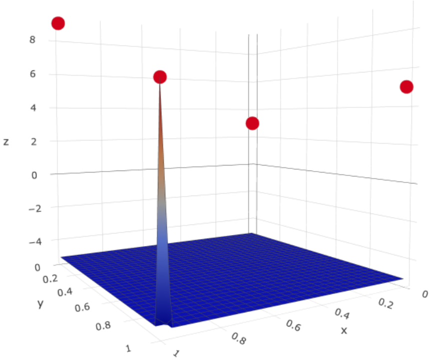
defined for all (z^t, λ^t) feasible to MP is a valid set of optimality cuts.

Proof.

Benders decomposition



Benders decomposition



Benders decomposition

Computationally catastrofic!

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LP relaxation cuts made it work

Benders decomposition

Computationally catastrofic!

LP relaxation cuts made it work

Plus some simmetry breaking

Results

$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{K}\rvert$	α^{FROM}	α^{TO}	CPLEX gap	CPLEX t	L-Shaped gap	L-Shaped gapR	L-Shaped gap50	L-Shaped t
50	200	0.2	0.2	0.0014	297.99	0.4747	21.5553	0.4747	1801.00
50	400	0.2	0.2	0.4151	1804.09	6.5981	59.3308	8.9622	1800.13
50	600	0.2	0.2	-	-	13.6751	52.5981	18.9363	1800.03
100	200	0.2	0.2	0.0000	221.91	0.0000	8.9488	-	16.47
100	400	0.2	0.2	-	-	0.4829	26.1941	0.8372	1801.67
100	600	0.2	0.2	-	-	9.2186	40.2920	9.2848	1805.22
200	400	0.2	0.2	-	-	0.0000	9.9094	-	229.96
200	600	0.2	0.2	-	-	0.0571	14.7626	0.0579	1801.39
50	200	0.2	0.8	0.0067	381.59	0.5159	20.9177	0.5159	1800.11
50	400	0.2	0.8	0.3913	1800.30	3.5957	43.2969	4.3436	1800.03
50	600	0.2	0.8	-	-	14.1098	63.4863	16.2944	1802.22
100	200	0.2	0.8	0.0000	199.66	0.1800	10.6250	0.1800	1800.13
100	400	0.2	0.8	-	-	0.1798	22.5452	0.3266	1800.02
100	600	0.2	0.8	-	-	9.1325	36.2728	9.8433	1808.65
200	400	0.2	0.8	-	-	0.0000	59.2478	-	165.06
200	600	0.2	0.8	-	-	0.1327	15.6153	0.4830	1800.02

Solutions

Table: Comparison of the solutions with and without dynamic pricing on the instances with 50 vehicles and 600 customers.

Distribution	Metric	With dynamic pricing	Without dynamic pricing
D1	Expected Profit [%]	100	81.78
	% of vehicles Relocated	26.0	10.0
	Min $\lvert\mathcal{R}(\xi)\rvert$	167	80
	Max $\lvert\mathcal{R}(\xi)\rvert$	195	107
	Expected % Requests satisfied	24	42
D2	Expected Profit [%]	100	66.06
	% of vehicles Relocated	22.0	2.0
	Min $\lvert\mathcal{R}(\xi)\rvert$	168	81
	Max $\lvert\mathcal{R}(\xi)\rvert$	187	105
	Expected % Requests satisfied	26	49
D3	Expected Profit [%]	100	65.05
	% of vehicles Relocated	18.0	6.0
	Min $\lvert\mathcal{R}(\xi)\rvert$	167	80
	Max $\lvert\mathcal{R}(\xi)\rvert$	195	107
	Expected % Requests satisfied	26	49
D4	Expected Profit [%]	100	66.36
	% of vehicles Relocated	10.0	0.0
	Min $\lvert\mathcal{R}(\xi)\rvert$	168	81
	Max $\lvert\mathcal{R}(\xi)\rvert$	187	105
	Expected % Requests satisfied	26	48

Take-aways

- A possible way of setting prices in CS services
- Complex integer stochastic program, but Benders decomposition went a long way
- Increases profits for the company
- Currently trying heuristics (with some help..)