

# Optimizing vessel fleet size and mix to support maintenance operations at offshore wind farms

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## Abstract

This paper considers the problem of determining the optimal vessel fleet to support maintenance operations at an offshore wind farm. We propose a two-stage stochastic programming (SP) model of the problem where the first stage decisions are what vessels to charter. The second stage decisions are how to support maintenance tasks using the chartered vessels from the first stage, given uncertainty in weather conditions and the occurrence of failures. To solve the resulting SP model we perform an ad-hoc Dantzig-Wolfe decomposition where, unlike standard decomposition approaches for SP models, parts of the second stage problem remain in the master problem. The decomposed model is then solved as a matheuristic by apriori generating a subset of the possible extreme points from the Dantzig-Wolfe subproblems. A computational study in three parts is presented. First, we verify the underlying mathematical model by comparing results to leading work from the literature. Then, results from in-sample and out-of-sample stability tests are presented to verify that the matheuristic gives stable results. Finally, we exemplify how the model can help offshore wind farm operators and vessel developers improve their decision making processes.

*Keywords:* Logistics; Offshore Wind; Fleet Size and Mix; Maintenance planning; Stochastic programming

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## 1. Introduction

The offshore wind industry is rapidly growing, and the installed offshore wind capacity in Europe has increased from less than 40 MW in 2000 to 15.78 GW by the end of 2017, with 11 new projects under construction with a planned total installed capacity of 2.9 GW (WindEurope, 2018). A major challenge for the industry is the high operating costs, making it difficult to operate profitably without government subsidies. The cost of energy needs to be reduced, and it is therefore essential to consider cost reductions in all parts of the value chain to make offshore wind

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a competitive alternative to other energy sources. One of the major cost components of operating an offshore wind farm is the cost of executing maintenance tasks, which is expected to account for up to 25% of the total life-cycle cost for an offshore wind farm (Phillips et al., 2013).

The most expensive resources in the operation and maintenance (O&M) phase of an offshore wind farm are the vessels needed to support the maintenance tasks. Offshore wind is a relatively new industry and new innovative vessel concepts, many of them fit-for-purpose, are being designed and launched to the market. The most resource-efficient vessel fleet to support maintenance tasks at a given offshore wind farm depends on many factors; e.g. size of wind farm, weather conditions at the site and distance to shore. A decision maker will hence benefit greatly from help in determining a resource-efficient vessel fleet for a given wind farm. Similarly, vessel designers and developers will benefit greatly from a tool that can analyse what type of wind farms their vessels are suited for, and to quantify the new vessel designs' economic advantages to the offshore wind supply chain.

The fleet size and mix problem for maintenance operations at offshore wind farms consists of determining an optimal fleet of vessels and bases to support all, or most of, the maintenance operations needed throughout the operational life of the wind farm. The maintenance operations may be divided into three main types of maintenance tasks. Corrective maintenance tasks are executed to repair and/or replace components that have failed on the turbines or other wind farm components. Preventive, or scheduled, maintenance tasks are executed with the aim of prolonging the lifetime of the wind turbines and other wind farm components, and to keep the number of failures at a reasonable level. When, and how often, such maintenance tasks are performed depend on the type of turbines, the wind farm operator's maintenance strategy, and the service intervals recommended by the turbine producer. Typical frequency is suggested to be 1-2 turbine visits every year with a major overhaul every 5 years (van Bussel et al., 2001). Condition based maintenance is a special case of preventive maintenance, and is performed based on the condition of the individual wind turbine, rather than a pre-determined frequency. However, the lack of reliable condition monitoring systems have prevented condition based maintenance from being widely used in the offshore wind industry thus far.

Whenever maintenance is performed on a wind turbine, the turbine must be shut down and does not produce any electricity. As a result, the operator suffers a loss in profit, often referred to as *downtime cost*. For preventive, and condition based, maintenance tasks downtime cost only accumulates during the execution of the task, i.e. when the turbine needs to be temporarily shut down due to maintenance work. For corrective maintenance tasks the downtime cost accumulates from the time of failure and until the maintenance task has been completed.

A fleet of maintenance vessels is needed to support the maintenance tasks. Vessels can be chartered either on long-term or short-term contracts, with fixed time charter costs depending on the type of vessel and the length and timing of the charter. The vessels have different characteristics

such as sailing speed, lifting capacity, technician capacity, access system, and weather limits. These characteristics determine which maintenance tasks a vessel can support and the vessel's *weather window*, i.e. the time interval on a given day during which the weather conditions continuously stay below the vessel's weather limits. A vessel may be used to support one single task at a time, or it may support several tasks in parallel, i.e. drop off a team of technicians at one turbine before moving to another turbine to drop off another team. Use of vessels incurs variable costs, mainly related to fuel costs.

Each chartered vessel operates in shifts that start when a vessel leaves its base and ends when it returns to the same base. A base has given capacities for vessels and technicians, and may be an onshore port, an offshore station, or mother vessel located close to, or within, the wind farm. A mother vessel is a vessel concept developed for the offshore wind industry consisting of a large vessel that functions as a base for smaller daughter vessels and provides accommodation for maintenance technicians. In addition, the mother vessel may support maintenance tasks, and may in some cases have a crane that enables it to support tasks that require heavy lift operations.

Similar problems to the one studied in this paper have previously been investigated by Halvorsen-Weare et al. (2013) and Gundegjerde et al. (2015). The first paper presents a deterministic model of the problem, while the second presents a three-stage SP formulation. Even though they study similar fleet size and mix problems, we present several key modelling improvements: 1) We do not put a limit on the number of time periods between a failure occurs and is fixed. This either severely limits the solution space if set too narrow or make the model unsolvable if set too wide. Especially for wind farms located in regions with harsh weather conditions, this may lead to the model suggesting an expensive fleet. E.g. if the weather conditions within the set of time periods allowed to fix a given failure only allow one (expensive) vessel type to operate, then this vessel type will be chartered regardless of its time charter cost. 2) We do not make the simplification of vessels having a number of units of time available each day which are assigned to maintenance tasks. This may result in overestimating the number of maintenance tasks a vessel can support each day and allows tasks to be left half-finished for several days. The model proposed in this paper considers possible sets of maintenance tasks a vessel can support in a given time period, and then assigns one set to each vessel each day of the planning horizon. Hence, we avoid overestimating the number of maintenance tasks supported by each vessel. 3) We use a circular time line to handle end-of-horizon effects. Not handling end-of-horizon effects in an appropriate way may result in chartering a very expensive vessel to execute maintenance tasks near the end of the planning horizon. In Sperstad et al. (2017) it was shown that the optimal vessel fleet is insensitive to changes in electricity prices and vessels' time charter rates. Hence, unlike Gundegjerde et al. (2015), we do not consider uncertainty in these parameters. In addition, we consider only a single wind farm, and do not allow vessels to move between different bases.

Using the classification scheme provided by Shafiee (2015) the problem studied in this paper is at the *tactical echelon* of offshore wind farm decision making, and within the category *maintenance support organization*. In addition to the two papers presented above, several papers have studied similar problems from this category. van de Pieterman et al. (2011) use an O&M Cost Estimator originally presented by Rademakers et al. (2008) to estimate the optimal number of access vessels needed to support maintenance at an offshore wind farm, while Dalgic et al. (2015c) use Monte-Carlo simulation to evaluate fleets consisting of different combinations of two types of crew transfer vessels (CTVs) in order to identify the fleet that minimizes the total O&M costs. Similar studies are presented by Dalgic et al. (2015d) and Dalgic et al. (2015b) where simulation models are used to evaluate charter strategies for jack-up vessels and mother ship concepts, respectively, at offshore wind farms. Other approaches to determining when to charter vessels are presented by Taylor & Jeon (2018) and Zhang & Zeng (2017). The former propose to use wave height probabilistic forecasts to make decisions on when to charter vessels, while the latter suggests a condition-based opportunistic preventive maintenance and spare parts provisioning policy to decide when to execute maintenance tasks. In addition to the studies mentioned above, several papers propose simulation models to evaluate the performance of the logistic system at offshore wind farms, e.g. Dalgic et al. (2015a), Hofmann & Sperstad (2013), and Endrerud et al. (2014). For a complete review of older papers considering maintenance support organisation we refer to Hofmann (2011), while different aspects of the installation phase of offshore wind farms that have been studied include port layout (Irawan et al., 2017b), cable routing (Fischetti & Pisinger, 2018), and installation planning and scheduling (Ursavas, 2017; Barlow et al., 2018).

An important decision when determining the optimal fleet size and mix of vessels to support maintenance at an offshore wind farm, is the daily routing of vessels, and scheduling of resources onboard the vessels, to support a set of available maintenance tasks. A mathematical model for this problem was first presented by Dai et al. (2015), while Stålhane et al. (2015) and Irawan et al. (2017a) present similar models that are decomposed by Dantzig-Wolfe decomposition into a master problem that assigns vessels to maintenance tasks, and one subproblem for each vessel where the routing and scheduling decisions are made. A further extension of the problem is presented by Schrottenboer et al. (2018) who include the daily allocation of technicians to multiple O&M bases that perform maintenance operations at multiple offshore wind farms. The problem is solved using an adaptive large neighborhood search heuristic that finds high quality solutions in short computing time.

Both Stålhane et al. (2015) and Irawan et al. (2017a) solve the decomposed problem by pre-generating all possible routes and schedules for each vessel on each day, followed by solving the master problem as a mixed integer program. We adopt a similar modelling approach, by applying Dantig-Wolfe decomposition to our two-stage stochastic model. However, since we need to solve this

problem for a much longer planning horizon and a higher number of vessels, we have made several simplifications. 1) The columns in the master problem does not represent routes and schedules that fix individual failures, but sets of maintenance tasks, hereafter referred to as *maintenance patterns*, that can be completed by a vessel type in a single time period. These tasks are then linked with individual failures in the master problem. This approach gives us considerably fewer possible columns, at the expense of having to use average, rather than exact, travel times within the wind farm. 2) We only consider one type of maintenance technicians. Using this approach enables us to model the day-to-day operations in greater detail than in previous works on the fleet size and mix problem, while keeping the mathematical model computationally tractable.

The problem of determining the size and composition of a fleet of vehicles is commonly referred to as the fleet size and mix problem (FSMP). There exists a large body of literature for land-based transportation, with a comprehensive review given by Hoff et al. (2010). However, land-based FSMPs differ considerably from the problem studied in this paper as the fleet is usually homogeneous, capital costs are much lower, and there is less uncertainty. Pantuso et al. (2014) conducted a literature survey of maritime FSMP, and identified 37 papers. Of these 27 solve the FSMP as a deterministic problem, while 10 consider some of the underlying uncertain elements. Of these 10 papers only Meng & Wang (2010), Alvarez et al. (2011), and Meng et al. (2012) explicitly treat uncertainty in their modelling approach. Since the survey was published, studies published by Bakkehaug et al. (2014), Pantuso et al. (2015), Patricksson et al. (2015), Pantuso et al. (2016), and Mørch et al. (2017) all present SP models to solve different versions of the maritime fleet size and renewal problem (MFSRP). The MFSRP differs from the classical FSMP in that an initial fleet already exists and the decisions are how to adapt this fleet over time. In addition, the deployment of the fleet is different since the vessels are used for transportation tasks rather than to support maintenance tasks.

In this paper we present a new two-stage stochastic programming (SP) model to find the optimal fleet of vessels to minimize the total O&M costs at an offshore wind farm using a maintenance strategy based on a combination of preventive and corrective maintenance tasks. The first stage decisions are which base(s) to use, and which vessels to charter on long-term and short-term contracts. In the second stage, the day-to-day deployment of the given vessel fleet is modelled to obtain an evaluation of operating costs for the vessels and the downtime costs of the wind farm. The deployment of the fleet is subject to uncertainty both in the number and timing of required maintenance tasks and the weather conditions at the wind farm site. The model presented in this paper has been developed in cooperation with Norwegian offshore wind operators Equinor (previously known as Statoil) and Statkraft, and has been used to provide decision support in the development phase of several offshore wind farms. The model and solution approach described in this paper has also been used to compare different access criteria for maintenance vessels (Sperstad

et al., 2014), compare the sensitivity of the optimal ranking of vessels (Sperstad et al., 2017), and benchmark a heuristic approach to the fleet size and mix problem (Halvorsen-Weare et al., 2017).

The contributions of this paper are: 1) A new two-stage SP model for the FSMP to support maintenance operations at offshore wind farms, where the operational use of each potential vessel fleet is modelled in much greater detail than in earlier studies. 2) An ad-hoc innovative solution method which consists of a Dantzig-Wolfe reformulation of the second stage problem. This reformulation allows us to solve the extensive form of the two-stage stochastic program without using classical decomposition methods such as L-Shaped or Dual Decomposition. 3) A matheuristic solution approach to the problem based on apriori generation of a subset of all feasible sets of maintenance tasks.

The remainder of this paper is organized as follows. Section 2 presents a mathematical formulation of the problem, followed by a description of the matheuristic solution approach in Section 3. The computational study is presented in Section 4, before we give some concluding remarks in Section 5.

## 2. Mathematical model

To find the optimal fleet of vessels to support maintenance tasks during a given planning horizon at an offshore wind farm, we have developed a two-stage SP model. The first stage decisions are which bases to use, and which vessels to charter both on long-term and short-term contracts. For short-term contracts we also need to determine what time periods to charter the vessels. The second stage decisions are to determine which maintenance tasks to support by what vessel in each time period of the planning horizon for a given realisation of the uncertain parameters. In the following we present a two-stage stochasting programming model of the problem. A complete list of all notation used in this Section can be found in Appendix B.

### 2.1. The first stage model

Let  $\mathcal{K}$  be the set of potential onshore and offshore bases where vessels start and end their operations. With each base  $k \in \mathcal{K}$  there is an associated operating cost  $C_k^B$ , that includes the depreciated investment cost for the base over the planning horizon. The parameter  $E_k$  is equal to one if base  $k$  must be part of a solution (specified by the user) and zero otherwise, while the parameter  $B_{k_1 k_2}$  is equal to one if both base  $k_1 \in \mathcal{K}$  and base  $k_2 \in \mathcal{K}$  can be part of an optimal solution (specified by the user), and zero otherwise.

Further, we have a set of vessel types  $\mathcal{V}$ , and subsets of vessel types  $\mathcal{V}_k$  that may operate from base  $k$ . Each vessel type  $v \in \mathcal{V}$  may be chartered both on long-term and short-term contracts, and we let  $C_v^{LT}$  be the long-term charter costs for vessels of type  $v$ . For vessels that are time chartered for a longer period than the planning horizon, the fixed costs will be the time charter costs associated with the planning horizon. For vessels that are not time chartered, but owned

by the wind farm operator, the fixed costs will be the depreciated investment cost for the vessel over the planning horizon. In addition,  $Q_{kv}^{MX}$  denotes the maximum number of vessels of type  $v$  that can be stationed at base  $k$ , and  $E_{kv}$  is the number of previously long-term chartered vessels of type  $v$  stationed at base  $k$ . The latter parameter can be used e.g. when a wind farm operator wants to expand the fleet, or add short-term chartered vessels for a maintenance campaign.

We define a set of charter periods  $\mathcal{T}$  in which vessels may be short-term chartered, and denote  $C_{vt}^{ST}$  and  $Q_{vt}^{MX}$  the cost, and maximum number of vessels, that may be chartered in charter period  $t \in \mathcal{T}$ , respectively. Given a planning horizon of  $\mathcal{P}$  discrete time periods (usually days or shifts), we assume that each  $t \in \mathcal{T}$  is a set of consecutive time periods in  $\mathcal{P}$ , all pairs  $t_1, t_2 \in \mathcal{T}$  are disjoint, and  $\bigcup \mathcal{T} = \mathcal{P}$ . A typical length of a charter period is one month.

There are three first stage decision variables. The variable  $\beta_k$  is equal to one if base  $k$  is used, and zero otherwise, while the variables  $\gamma_{kv}^{LT}$  and  $\gamma_{kvt}^{ST}$  are the number of vessels of type  $v$ , stationed at base  $k$ , chartered on long-term contracts and short-term contracts in time period  $t$ , respectively. We thus assume that a vessel is stationed at the same base throughout the planning horizon. Further, we let  $\tilde{\xi}$ , which will be formally introduced in Section 2.2, represent the uncertain parameters of the problem. Finally, let  $Q(\gamma^{LT}, \gamma^{ST}, \tilde{\xi})$  be the *recourse function* which represents the total cost of executing maintenance tasks at the wind farm, given a set of first stage decisions and a realisation  $\xi$  of  $\tilde{\xi}$ . Using this notation the first stage model can be defined as follows:

$$\min \sum_{k \in \mathcal{K}} C_k^B \beta_k + \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}_k} C_v^{LT} \gamma_{kv}^{LT} + \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}_k} \sum_{t \in \mathcal{T}} C_{vt}^{ST} \gamma_{kvt}^{ST} + E_{\tilde{\xi}}[Q(\gamma^{LT}, \gamma^{ST}, \tilde{\xi})] \quad (1)$$

subject to:

$$\gamma_{kv}^{LT} + \gamma_{kvt}^{ST} \leq Q_{kv}^{MX} \beta_k, \quad k \in \mathcal{K}, v \in \mathcal{V}_k, t \in \mathcal{T}, \quad (2)$$

$$\beta_{k_1} + \beta_{k_2} \leq 1, \quad k_1, k_2 \in \mathcal{K} \mid B_{k_1 k_2} \neq 1, \quad (3)$$

$$\beta_k \geq E_k, \quad k \in \mathcal{K}, \quad (4)$$

$$\gamma_{kv}^{LT} \geq E_{kv}, \quad k \in \mathcal{K}, v \in \mathcal{V}_k, \quad (5)$$

$$\sum_{k \in \mathcal{K}} \gamma_{kvt}^{ST} \leq Q_{vt}^{MX}, \quad v \in \mathcal{V}, t \in \mathcal{T}, \quad (6)$$

$$\beta_k \in \{0, 1\}, \quad k \in \mathcal{K}, \quad (7)$$

$$\gamma_{kv}^{LT} \in Z^+, \quad k \in \mathcal{K}, v \in \mathcal{V}_k, \quad (8)$$

$$\gamma_{kvt}^{ST} \in Z^+, \quad k \in \mathcal{K}, v \in \mathcal{V}_k, t \in \mathcal{T}. \quad (9)$$

The first term of the Objective function (1) is the total cost of operating the bases, including capital and operational costs. The second and third term are the time charter costs, or capital costs, associated with vessels that are in the fleet the whole planning horizon and that are time chartered for shorter parts of the planning horizon, respectively. Finally, the fourth term is the expected

cost of using a given fleet to support maintenance tasks at the offshore wind farm. Constraints (2) state that the total number of vessels operating from a base in any given short-term time charter period cannot exceed the capacity at that base, while Constraints (3) ensure that at most one of two non-compatible bases are used. Constraints (4) and (5) ensure that existing bases and vessels are included in the optimal solution, and Constraints (6) limit the number of vessels time-chartered in each charter period to the maximum number available. Finally, Constraints (7)–(9) set binary and integral requirements on the variables.

## 2.2. The second stage model

The second stage problem determines how to support a set of maintenance tasks with a given fleet so that the total O&M costs of a planning horizon of  $\mathcal{P}$  time periods are minimized. To solve this problem we need to model the day-to-day operations of the given fleet, i.e. which maintenance tasks each vessel supports in each time period. This includes deciding which maintenance tasks are performed in each time period, and which vessel performs which tasks.

To formulate the decomposed model of the second stage problem we need to define some additional notation. Let  $\mathcal{M}$  be a set of maintenance task categories, and let  $T_i^M$  be the time it takes to complete a maintenance task of category  $i \in \mathcal{M}$ . The set  $\mathcal{M}$  is divided into one set of preventive maintenance task categories,  $\mathcal{M}^P$ , and one set of corrective maintenance task categories,  $\mathcal{M}^C$ . Preventive maintenance task categories may be a yearly maintenance or major overhaul, and for each preventive maintenance task category  $i \in \mathcal{M}^P$ , there are  $A_i$  tasks that must be completed during the planning horizon. Corrective maintenance tasks categories include minor repair, major repair, minor replacement, and major replacement (Dinwoodie et al., 2015). The set of corrective maintenance task categories can be further separated into a set of categories that require vessels with heavy lift capabilities,  $\mathcal{M}^L$ , and categories that only require a number of technicians,  $\mathcal{M}^T$ . It is assumed that all maintenance task categories in  $\mathcal{M}^T \cup \mathcal{M}^P$  can be performed within a single time period by any vessel, while those in  $\mathcal{M}^L$  may take several time periods, can only be performed by vessels of type  $\mathcal{V}^L \subset \mathcal{V}$ , and requires the vessel to be present for the duration of the execution of the task. We assume that all maintenance task categories only needs the support of one vessel.

When modelling the day-to-day operations of the given fleet we consider uncertainty both in the weather conditions at the offshore wind farm, and in the occurrence of failures that require maintenance. Let  $\tilde{C}_p^D$  be the stochastic downtime cost associated with not producing electricity in time period  $p$ ,  $\tilde{F}_{iq}$  the stochastic number of failures that require a corrective maintenance task of category  $i \in \mathcal{M}^C$  in time period  $q$ , and  $\tilde{T}_{vp}^{max}$  the maximum amount of time a vessel of type  $v$  can operate in period  $p$ , calculated as the stochastic minimum between the length of a time period (typically the length of a working shift), and the length of the vessel’s weather window in period  $p$ . We assume that vessels only use one weather window per time period, and that they choose the largest one. Let  $\tilde{\xi}$  be the collection of the stochastic parameters, that is:  $\tilde{\xi} =$



$(\tilde{C}_1^D, \dots, \tilde{C}_{|\mathcal{P}|}^D, \tilde{F}_{11}, \dots, \tilde{F}_{|\mathcal{M}|, |\mathcal{P}|}, \tilde{T}_{11}^{max}, \dots, \tilde{T}_{|\mathcal{V}|, |\mathcal{P}|}^{max})$ . We assume that the probability distributions of  $\tilde{\xi}$  are known. Based on  $\tilde{T}_{vp}^{max}$ , we may also calculate the number of time periods a maintenance task  $i \in \mathcal{M}^L$  take,  $\tilde{T}_{vpi}^L$ , as

$$\tilde{T}_{vpi}^L = \min\{x \in Z^+ \mid \sum_{p'=p}^{p+x-1} \tilde{T}_{vp'}^{max} \geq T_i^M\}.$$

To avoid end-of-horizon effects with failures appearing late in a planning horizon, we model the planning horizon as a circle rather than as a line, and thus a failure may be repaired in an earlier time period than it occurs. In these cases repairing the failure in a given time period incurs the same downtime cost as if it happened in the same time period in the previous planning horizon. This is illustrated in Figure 1. If a failure that appeared in time period  $q$  is repaired in time period  $p$ , we denote the downtime cost as  $\tilde{C}_{qp}^D$  and it is calculated as:

$$\tilde{C}_{qp}^D = \begin{cases} \sum_{p'=q}^p \tilde{C}_{p'}^D, & \text{if } q \leq p, \\ \sum_{p'=q}^{|\mathcal{P}|} \tilde{C}_{p'}^D + \sum_{p'=1}^p \tilde{C}_{p'}^D, & \text{otherwise.} \end{cases}$$

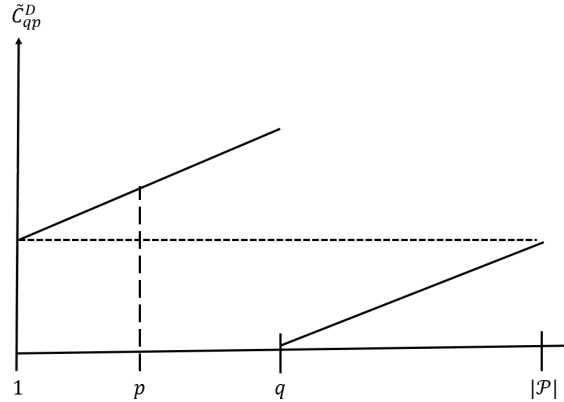


Figure 1: Illustration of how the downtime cost of a failure occurring in time period  $q$  increases over time until it is fixed in time period  $p$ .

To model the use of each vessel in each time period, we use *maintenance patterns* which consists of sets of maintenance tasks that a given vessel type  $v$  is able to perform within a single time period when operating out of base  $k$ . Let  $\mathcal{W}_{kv}$  indexed by  $w$  be the set of feasible maintenance patterns for vessel type  $v$  operating from base  $k$ , and let  $A_{iw}$  be the number of maintenance tasks of category  $i$  that is performed by pattern  $w$ . Further, let  $C_w$  and  $T_w$  be the cost and duration of maintenance pattern  $w$ , respectively. Determining the cost  $C_w$ , the time  $T_w$ , and checking the feasibility, for a given set of values for the  $A_{iw}$  parameters, may be modelled as a one-to-one pick-up and delivery problem. The mathematical formulation of this problem is given in Section 3.1.

In the mathematical model of the second stage problem we use the variables  $y_{qpi}$  to denote the number of failures requiring a maintenance task of category  $i$  occurring in time period  $q$  that is repaired in time period  $p$ , while  $z_i$  denotes to the number of maintenance tasks of category  $i$  that

is not executed during the planning horizon. Each task not executed during the planning horizon, is penalized with an artificial penalty cost  $C_i^P$ . In addition,  $\lambda_{kvpw}$  and  $u_{kvp_i}$  are integer variables denoting how many vessels of type  $v$  operating from base  $k$  use maintenance pattern  $w$  or perform maintenance task  $i \in \mathcal{M}^L$  in time period  $p$ , respectively. Given specific values  $\hat{\gamma}^{LT}$  and  $\hat{\gamma}^{ST}$ , for  $\gamma^{LT}$  and  $\gamma^{ST}$ , respectively, and a specific realization  $\xi$  of  $\tilde{\xi}$ , the second stage problem can be stated as follows:

$$Q(\hat{\gamma}^{LT}, \hat{\gamma}^{ST}, \xi) = \min \sum_{q \in \mathcal{P}} \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{M}^C} \tilde{C}_{qp}^D y_{qpi} + \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}_k} \sum_{w \in \mathcal{W}_{kv}} \sum_{p \in \mathcal{P}} \tilde{C}_p^D A_{iw} \lambda_{kvpw} + \sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}_k} \sum_{w \in \mathcal{W}_{kv}} \sum_{p \in \mathcal{P}} C_w \lambda_{kvpw} + \sum_{i \in \mathcal{M}} C_i^P z_i, \quad (10)$$

subject to:

$$\sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}_k} \sum_{p \in \mathcal{P}} \sum_{w \in \mathcal{W}_{kv}} A_{iw} \lambda_{kvpw} + z_i = A_i, \quad i \in \mathcal{M}^P, \quad (11)$$

$$\sum_{p \in \mathcal{P}} y_{qpi} + z_i = \tilde{F}_{iq}, \quad q \in \mathcal{P}, i \in \mathcal{M}^C, \quad (12)$$

$$\sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}} \sum_{w \in \mathcal{W}_{kv}} A_{iw} \lambda_{kvpw} - \sum_{q \in \mathcal{P}} y_{qpi} = 0, \quad p \in \mathcal{P}, i \in \mathcal{M}^C, \quad (13)$$

$$\sum_{k \in \mathcal{K}} \sum_{v \in \mathcal{V}^L} u_{kv(p - \tilde{T}_{vp_i}^L + 1)i} - \sum_{q \in \mathcal{P}} y_{qpi} = 0, \quad p \in \mathcal{P}, i \in \mathcal{M}^L, \quad (14)$$

$$\sum_{w \in \mathcal{W}_{kv}} \lambda_{kvwq} + \sum_{i \in \mathcal{M}^L} \sum_{p=q}^{q + \tilde{T}_{vp_i}^L} u_{kvp_i} \leq \hat{\gamma}_{kv}^{LT} + \hat{\gamma}_{kvt}^{ST}, \quad k \in \mathcal{K}, v \in \mathcal{V}, t \in \mathcal{T}, q \in t, \quad (15)$$

$$(T_w - \tilde{T}_{vp}^{max}) \lambda_{kvpw} \geq 0, \quad v \in \mathcal{V}_k, w \in \mathcal{W}_{kv}, p \in \mathcal{P}, \quad (16)$$

$$\lambda_{kvpw} \in Z^+, \quad k \in \mathcal{K}, v \in \mathcal{V}_k, w \in \mathcal{W}_{kv}, p \in \mathcal{P}, \quad (17)$$

$$y_{qpi} \in Z^+, \quad p, q \in \mathcal{P}, i \in \mathcal{M}^C, \quad (18)$$

$$u_{kvp_i} \in Z^+, \quad k \in \mathcal{K}, v \in \mathcal{V}_k, p \in \mathcal{P}, i \in \mathcal{M}^L, \quad (19)$$

$$z_i \in Z^+, \quad i \in \mathcal{M}. \quad (20)$$

The Objective function (10) minimizes the total cost of maintenance at the wind farm over the planning horizon. The first terms represent the downtime costs associated with corrective maintenance tasks, while the second term gives preventive downtime costs and the total transportation costs of applying a given maintenance pattern in a given time period. The third term is a penalty cost for not performing a maintenance task within the planning horizon. Constraints (11) and (12) ensure that all preventive and corrective maintenance tasks are either executed during the planning horizon or given a penalty in the objective function. Further, Constraints (13) and (14) map each corrective maintenance task to a specific failure, while Constraints (15) make sure that the number of maintenance patterns used and lifting maintenance tasks undertaken in a given

time period do not exceed the number of vessels available in that period. Finally, Constraints (16) determines which periods a given maintenance pattern can be executed, and Constraints (17)–(20) set integral requirements on the variables.

### 2.3. Vessels that can stay offshore for multiple periods

Some vessels, such as accommodation vessels and jack-up vessels, stay offshore for multiple time periods. To correctly model these vessels' behaviour, an artificial base, representing the wind farm, is added to the model and used as a base by these vessels. To ensure that these vessel types periodically have to return to base to re-supply and change crew, we may remove some time periods, e.g. one period every two weeks, to ensure that the vessels are not available for maintenance work on these days. The additional travel costs of these return trips are not explicitly included in the model, but could be added to the vessel charter cost ( $C_v^{LT}$  or  $C_{vt}^{ST}$ ).

Another class of vessels that requires more advanced modelling are the mother vessel concepts, where one large vessel accommodates several smaller daughter vessels. In this case the mother vessel is modelled as a base located at the wind farm, and both the mother vessel itself and all daughter vessels are modelled as vessels belonging to this base. To avoid duplicating the charter cost for a mother vessel, all time charter costs are added to the base, and all vessels accommodated by that base have a time charter cost of 0.

## 3. Solution methodology

To solve the model presented in Section 2 we describe the random variables  $\tilde{\xi}$  by means of a set  $\mathcal{S}$  of representative scenarios, each scenario representing one realisation of downtime costs, failures, and weather windows for the entire planning horizon. Consequently, we can replace  $E_{\tilde{\xi}}[Q(\gamma^{LT}, \gamma^{ST}, \tilde{\xi})]$  with  $\sum_{s \in \mathcal{S}} p_s Q(\gamma^{LT}, \gamma^{ST}, \xi_s)$  in (1) where  $p_s$  is the probability of scenario  $s$ , and  $\xi_s = (C_{1s}^D, \dots, C_{|\mathcal{P}|s}^D, F_{11s}, \dots, F_{|M|,|\mathcal{P}|s}, T_{11s}^{max}, \dots, T_{|V|,|\mathcal{P}|s}^{max})$  is the realisation of  $\tilde{\xi}$  in scenario  $s$ . This entails solving the extensive form of the deterministic equivalent problem.

### 3.1. Feasible maintenance patterns

The mathematical model presented in Section 2 relies on the generation of all feasible maintenance patterns for a given combination of bases and vessels. For each maintenance task category,  $i \in \mathcal{M}^T$ , let  $T_{vi}$  denote the average travel time associated with maintenance task  $i$  for a vessel of type  $v$  including the transfer of technicians and equipment to a turbine. The maximum length of a shift for vessel  $v$  stationed at base  $k$  is given by  $T_{kv}^{max}$ . Further, we let  $F_v$  be the fuel consumption per time unit for vessels of type  $v$  and  $C^F$  the fuel cost. Finally, we denote  $M_v$  as the maximum number of technicians that can be onboard vessel type  $v$ .

For a given combination of base  $k$  and vessel  $v$ , we may model all feasible maintenance patterns as a one-to-one pick-up and delivery problem (Berbeglia et al., 2007). Let  $G = (\mathcal{N}, \mathcal{A})$  be a graph,

where  $\mathcal{N}$  is the set of nodes and  $\mathcal{A}$  is the set of arcs. Let each node in  $\mathcal{N}$  be denoted  $(i, m)$  where  $i \in \{1, \dots, 2N\}$  and  $m \in \{1, \dots, U_{iv}\}$ , where  $N = |\mathcal{M}^T \cup \mathcal{M}^P|$  and  $U_{iv}$  is an upper bound on the number of maintenance tasks of category  $i$  that vessel type  $v$  can complete in one time period. The set of nodes can be divided into two disjoint sets: the set of delivery nodes  $(i, m)$  are defined for  $i \in \mathcal{M}^T \cup \mathcal{M}^P$ , where  $L_i$  technicians disembark to perform the maintenance task, and the set of pick-up nodes  $(i + N, m)$  where the technicians dropped off at node  $(i - N, m)$  are collected by the vessel after completing the task (for modelling purposes  $L_{(N+i)} = -L_i$ ). In addition, the set of nodes  $\mathcal{N}$  contains two nodes,  $(o, 1)$  and  $(d, 1)$  representing the base where the vessel begins and ends its shift, with  $T_{vo}$  giving the total travel time for a return trip from the base to the wind farm. The set of arcs  $\mathcal{A}$  consists of arcs between all node pairs, with the following exceptions: there are no arcs entering node  $(o, 1)$ , no arcs leaving node  $(d, 1)$ , and no arcs from a pick-up node to the corresponding delivery node.

Figure 2 shows an example of such a graph for a vessel type with a capacity of 6 technicians that may stay offshore up to 12 hours. This vessel type may perform two maintenance task categories ( $\mathcal{M}^T = \{1, 2\}$ ) which require 4 and 3 technicians, respectively, and have completion times of 8 hours. Given this input data, we obtain upper bounds  $U_{1v} = 1$  and  $U_{2v} = 2$  on the number of maintenance tasks that can be performed in a single time period. Note that for ease of exposition, we have used edges rather than arcs, when there are arcs in both directions between two nodes. A possible route for the vessel through the graph is:  $(o, 1) - (2, 1) - (2, 2) - (4, 1) - (4, 2) - (d, 1)$ , meaning that the vessel has performed two maintenance tasks of type 2, and none of type 1.

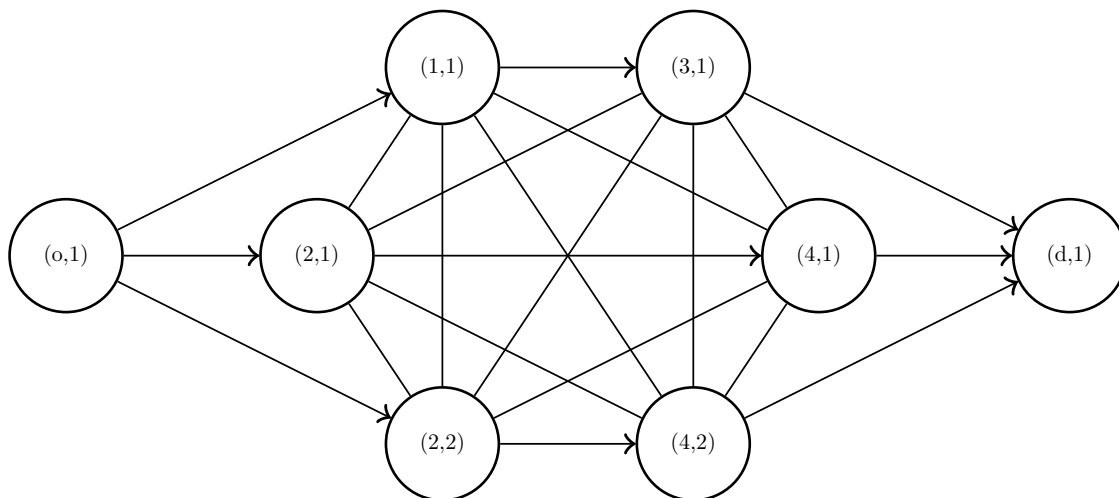


Figure 2: Small example graph for a problem with two maintenance task categories, where the vessel may perform a maximum of 1 and 2 tasks, respectively, in a given time period.

We have formulated an arc-flow model containing three sets of variables. The variables  $x_{imjn}$  are equal to one if the vessel traverses arc  $((i, m), (j, n))$ , and zero otherwise,  $t_{im}$  denotes the time node  $(i, m)$  is visited and  $l_{im}$  denotes the number of technicians on the vessel when leaving node

$(i, m)$ , respectively.

$$C_w = \min \sum_{((i,m),(j,n)) \in \mathcal{A}} T_{vi} F_v C^F x_{imjn}, \quad (21)$$

subject to:

$$\sum_{m=1}^{U_{iv}} \sum_{(j,n) \in \mathcal{N}} x_{imjn} = A_{iw}, \quad i \in \mathcal{M}^T \cup \mathcal{M}^P, \quad (22)$$

$$\sum_{(j,n) \in \mathcal{N}} x_{o1jn} = 1, \quad (23)$$

$$\sum_{(i,m) \in \mathcal{N}} x_{imjn} - \sum_{(i,m) \in \mathcal{N}} x_{jnim} = 0, \quad (j, n) \in \mathcal{N}, \quad (24)$$

$$\sum_{(i,m) \in \mathcal{N}} x_{imd1} = 1, \quad (25)$$

$$\sum_{(i,m) \in \mathcal{N}} x_{imjn} \leq 1, \quad (j, n) \in \mathcal{N}, \quad (26)$$

$$\sum_{(i,m) \in \mathcal{N}} x_{imjn} - \sum_{(i,m) \in \mathcal{N}} x_{im(N+j)n} = 0, \quad (j, n) \in \mathcal{N} \mid j \leq N, \quad (27)$$

$$0 \leq l_{im} \leq M_v - L_i, \quad (i, m) \in \mathcal{N} \mid i \leq N, \quad (28)$$

$$L_i \leq l_{im} \leq M_v, \quad (i, m) \in \mathcal{N} \mid i > N, \quad (29)$$

$$(l_{im} + L_j - l_{jn})x_{imjn} = 0, \quad ((i, m), (j, n)) \in \mathcal{A}, \quad (30)$$

$$t_{d1} \leq T_{kv}^{max}, \quad (31)$$

$$(t_{im} + T_{iv} - t_{jn})x_{imjn} \leq 0, \quad ((i, m), (j, n)) \in \mathcal{A}, \quad (32)$$

$$(T_i^M + T_{iv}) \sum_{(j,n) \in \mathcal{N}} x_{imjn} \leq t_{(N+i)m} - t_{im}, \quad (i, m) \in \mathcal{N} \mid i \leq N, \quad (33)$$

$$x_{imjn} \in \{0, 1\}, \quad ((i, m), (j, n)) \in \mathcal{A}. \quad (34)$$

The Objective function (21) minimizes the transportation costs, while Constraints (22) state that the number of each maintenance task category performed must match  $A_{iw}$ . Constraints (23)–(26) ensure that the route is continuous from node  $(o, 1)$  to node  $(d, 1)$  in the problem network and that each node is visited at most once. Further, Constraints (27) are the pairing constraints, ensuring that either both the pick-up and delivery nodes of a given pair is visited, or none of them. The number of technicians on the vessel when leaving a given node is kept track of by Constraints (28)–(30), while Constraints (31)–(32) keep track of the time at which each node is visited. Finally, Constraints (33) ensure that the time between the visit to a delivery node and the corresponding pick-up node is sufficient for the maintenance task to be performed, while Constraints (34) set binary requirements on the  $x$ -variables.

### 3.2. A heuristic for generating maintenance patterns

The number of feasible maintenance patterns may for many real instances of the problem be too large to generate efficiently. We therefore elect to use a heuristic maintenance pattern algorithm that creates a subset of all feasible paths through the network defined on graph  $G = (\mathcal{N}, \mathcal{A})$ , by not allowing vessels to visit a delivery node directly after a pick-up node, unless all the technicians are on-board the vessel. This means that all paths generated have repeated sequences of consecutive visits to delivery nodes, followed by a sequence of visits to the corresponding pick-up nodes. In addition to reduce the number of maintenance patterns in the model, this rule also mimics how many offshore wind operators use their vessels today. Thus, an additional advantage is that it avoids solutions to the second stage problem that deploy the available vessels better than manual planners would.

To generate maintenance patterns, we have used a labeling algorithm, following the approach described by Irnich & Desaulniers (2005), and applied to pickup and delivery problems by, among others, Røpke & Cordeau (2009) and Stålhane et al. (2012). They present labeling algorithms as a dynamic programming approach that may be applied to find the set of Pareto-optimal paths to a shortest path problem with resource constraints (SPPRC). In their approach, a label is used to represent a (partial) path from a depot to a node  $i$ , together with information regarding the accumulation of resources along the path. Each resource is required to stay within given limits, referred to as resource windows, at each node along the path. Labels are extended along arcs in the problem-defining network creating new labels where the resources are updated according to resource extension functions (REF) and checked for feasibility with respect to the resource windows. Any label extended to the end node of the graph represents a complete feasible path through the network. To avoid generating sub-optimal paths, a dominance step is introduced to remove labels whose extensions cannot become Pareto-optimal.

A pseudo-code for the labeling algorithm is given by Algorithm 1. Let  $U$  be the set of unprocessed labels, initially only containing the label  $L_0$  representing a path that consists of only the vessel type's origin node,  $(o, 1)$ .  $\mathcal{L}$  is then the set of processed, non-dominated labels. While there are labels in  $U$ , the *removeFirst*( $U$ )-function removes one label from  $U$  according to some criteria. The label, representing a path ending at node  $(i, m)$ , is extended along all arcs  $((i, m), (j, n)) \in \mathcal{A}$ , creating new labels,  $L'$ . If the new label is resource feasible, it is added to  $U$ . If the new label  $L'$  does not have any ongoing maintenance tasks, it is checked for dominance against all other labels with no ongoing maintenance tasks. Any labels in  $\mathcal{L}$  that are dominated by  $L'$  are removed from both  $\mathcal{L}$  and  $U$ . After all non-dominated labels have been produced, we go through each time period in each scenario, and create one maintenance pattern for each combination, given that there is a sufficient weather window to perform the maintenance pattern.

In the following subsections we define the data stored in a label, the resource extension functions,

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**Algorithm 1** Pseudo-code of labeling algorithm for vessel  $v$  at base  $k$ 


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Input: graph  $G_v = (\mathcal{N}_v, \mathcal{A}_v)$   
 $U = \{L_0\}$   
**while**  $U \neq \emptyset$  **do**  
    $L = \text{removefirst}(U)$   
   let  $(i, m)$  be the last node of the path represented by label  $L$   
   **for** each node  $(j, n) : ((i, m), (j, n)) \in \mathcal{A}$  **do**  
     create new label  $L'$  extending  $L$  to node  $(j, n)$   
     **if**  $L'$  is resource feasible **then**  
        $U = U \cup \{L'\}$   
     **end if**  
     **if**  $\mathcal{O}(L') = \emptyset$  **then**  
       remove all labels in  $\mathcal{L}$  and  $U$  that are dominated by  $L'$   
        $\mathcal{L} = \mathcal{L} \cup \{L'\}$   
     **end if**  
**end for**  
**end while**  
 return  $\mathcal{L}$

---

as well as the resource windows, and the dominance criterion applied.

### 3.2.1. Labels

Each label contains the following data:

1.  $\eta$  - the last node on the partial path
2.  $t_{lo}$  - the time node  $(l, o) \in \mathcal{N}$  is visited
3.  $c$  - the accumulated cost
4.  $\Delta$  - the set of maintenance tasks started (and possibly completed)
5.  $\mathcal{O}$  - the set of active maintenance tasks, i.e. started, but not completed

The notation  $c(L)$  is used to refer to the accumulated cost of label  $L$  and similar notation is used also for the other data (i.e.,  $\eta(L)$ ,  $t_{lo}(L)$ ,  $\Delta(L)$ , and  $\mathcal{O}(L)$ ). Note that we do not explicitly need to keep track of the number of technicians used, since this is implicitly given by the set of active maintenance tasks.

### 3.2.2. Label extension

When extending a label  $L$  with  $\eta(L) = (i, m)$  along arc  $((i, m), (j, n))$ , we create a new label  $L'$ . We let  $\eta(L') = (j, n)$  and let  $f_{imjn}^c(L)$  denote the resource extension function of the cost resource  $c$ , with similar functions for each of the resources stored in a label. The resource functions are defined as follows:

$$f_{imjn}^{t_{lo}}(L) = \begin{cases} t_{lo}(L), & \text{if } (l, o) \neq (j, n), \\ t_{lo}(L) + T_{vi}, & \text{if } (l, o) = (j, n) \text{ and } 1 \leq j \leq N, \\ \max\{t_{im}(L) + T_{vi}, t_{(j-N, n)}(L) + T_{j-N}^M + T_{v(j-N)}\}, & \text{otherwise,} \end{cases} \quad (35)$$

$$f_{imjn}^c(L) = c(L) + T_{vi}F_vC^F, \quad (36)$$

$$f_{imjn}^\Delta(L) = \begin{cases} \Delta(L) \cup \{(j, n)\} & \text{if } 1 \leq j \leq N, \\ \Delta(L) & \text{otherwise,} \end{cases} \quad (37)$$

$$f_{imjn}^\mathcal{O}(L) = \begin{cases} \mathcal{O}(L) \cup \{(j, n)\} & \text{if } 1 \leq j \leq N, \\ \mathcal{O}(L) \setminus \{(j - N, n)\} & \text{if } N + 1 \leq j \leq 2N. \end{cases} \quad (38)$$

The REF described in Equation (35) copies the time every node  $(o, l)$  is visited on the path, and calculated the time node  $(j, n)$  is visited. The REF described in Equation (36) adds the fuel cost of traveling to the next turbine. Further, the REF described in Equation (37) describe which delivery nodes in the network have been visited, and Equation (38) stores which delivery nodes have been visited where the corresponding delivery node has not.

An extended label  $L'$  with  $\eta(L') = (j, n)$  is considered resource feasible if:

$$t_{jn}(L') \leq T_{kv}^{max}, \quad (39)$$

$$\sum_{(i,m) \in \mathcal{O}(L')} L_i \leq M_v, \quad (40)$$

$$0 < j \leq N \rightarrow 0 < i \leq N \vee \mathcal{O}(L) = \emptyset \quad (41)$$

and one of the following hold:

$$0 < j \leq N \wedge (j, n) \notin \Delta(L) \wedge (n = 1 \vee (j, n - 1) \in \Delta(L)) \quad (42)$$

$$N < j \leq 2N \wedge (j - N, n) \in \mathcal{O}(L) \quad (43)$$

$$j = d \wedge \mathcal{O}(L) = \emptyset \quad (44)$$

Constraint (39) ensures that the total time spent plus the time it takes to get back to base is less than the maximum time of a maintenance pattern, while Constraint (40) states that the number of technicians currently working on maintenance tasks is less than or equal to the number of technicians on the vessel. Further, Constraint (41) states that if node  $(j, n)$  is a delivery node, then either node  $(i, m)$  is also a delivery node, or the set of ongoing maintenance tasks is empty. This constraint enforces the heuristic sequencing of tasks, and without this restriction, all feasible paths would be generated. Constraint (42) ensures that a delivery node  $(j, n)$  is visited only once, and that it is only visited if all delivery nodes for the same maintenance task with a lower index has been visited. The last part is added to avoid symmetry. Finally, Constraint (43) states that a pick-up node can only be visited if the corresponding delivery node has been visited, and Constraint (44) makes sure that the vessel only returns to base if all maintenance tasks have been completed.

### 3.2.3. Heuristic Dominance criterion

To limit the number of labels generated by the algorithm we employ a heuristic dominance criterion that remove labels that are considered inferior. The dominance criterion used to remove



dominated labels is the following:

A label  $L_i$  dominates  $L_j$  if:

1.  $\mathcal{O}(L_i) = \mathcal{O}(L_j) = \emptyset$
2.  $t(L_i)_{\eta(L_i)} \leq t(L_j)_{\eta(L_j)}$
3.  $\mathcal{V}(L_i) \geq \mathcal{V}(L_j)$

Thus, if label  $L_i$  represents a partial path that has supported more maintenance tasks in less time than  $L_j$  and both have no active maintenance tasks, label  $L_j$  is discarded. Note that we do not consider the cost of the two labels here, and assume it is always better to perform more maintenance tasks.

#### 4. Computational Study

The computational study presented in this section is divided into four parts. The first part provides an overview of the test instances, including how we generate scenarios for the second stage. The second part verifies that the solution approach gives reasonable estimates of the O&M costs of the wind farm, and the third part tests the stability and behaviour of the SP model. Finally, the fourth part provides some examples of how the model may provide valuable decision support to a wind farm operator or vessel producer.

The mathematical model is solved using the commercial optimization software Xpress that can be used to solve e.g. mixed integer linear programs. To reduce the computational complexity, we relax the integer requirements on the  $\lambda$ -variables which represent the number of times a given maintenance pattern is performed in a given period. All tests have been run on a HP bl68c G7 computer, with four 2.2 GHz AMD Opteron 6274 16 core processors and 128 GB of RAM, and the optimality gap was set to 1 %.

##### 4.1. Test instances

To test the SP model, we have used the reference wind farm presented in Dinwoodie et al. (2015). The main input data from their reference case can be found in Tables A.1–A.3, and these are used for the verification part of the model testing. However, since this reference case includes only one type of crew transfer vessel, we introduce additional vessel types described in Sperstad et al. (2017). The additional vessels are presented in Table A.4, and include a fast, but expensive, surface effect ship, one accommodation vessel that can stay offshore for multiple periods, and a mother vessel concept with two daughter vessels. We assume that all the additional vessel types can support the same set of maintenance tasks as a CTV from the reference case. In order to correctly model the small accommodation vessel and the mother vessel, we have added an artificial base, and a mother vessel base, both located at the wind farm. The cost of the mother vessel is

added to  $C_k^B$  rather than  $C_{kv}^{LT}$ , to prevent the daughter vessels being used without having chartered the mother vessel.

#### 4.2. Scenario generation

In the computational study we let each second stage problem represent a planning problem for one maintenance year. For each scenario we draw one random year from the set of weather data, and assume the realisation of the weather in the scenario is exactly equal to the weather of that year. Historical weather data is taken from BSH (2012) and includes wave height and wind speed with a granularity of one hour. Let  $H_{tps}^S$  and  $W_{tps}$  be the wave height and wind speed at hour  $t$  in time period  $p$  in the year of weather data used in scenario  $s$ , respectively. Further, let  $H_v^S$  be the wave height limit of vessel  $v$ , let  $E^{price}$  be the electricity price used and let  $Pow()$  be a function giving the power output of a wind turbine at a given wind speed. Using this notation the values of  $T_{vps}^{max}$  and  $C_{ps}^D$  can be calculated as follows:

$$T_{vps}^{max} = \max\{x \in Z^+ | \exists t \in \{1, \dots, T_{kv}^{max} - x + 1\}, \forall \tau \in \{t, \dots, t + x - 1\}, H_{\tau p}^S \leq H_v^S\}, \quad (45)$$

$$C_{ps}^D = \sum_{t=1}^{24} E^{Price} Pow(W_{tps}). \quad (46)$$

Equation (45) states that  $T_{vps}^{max}$  is the largest number of consecutive hours within a time period where the wave height is lower than the vessel's limit, while Equation (46) states that  $C_{ps}^D$  is the sum over 24 hours of the electricity production at each hour multiplied with the electricity price.

The corrective maintenance tasks that need to be executed at the offshore wind farm during the planning horizon are the result of failures at individual wind turbines. The second stage problem treats the realisation of future failures as known at the planning stage. The number of failures  $F_{iqs}$  must be generated for each scenario  $s$ , time period  $q$ , and corrective maintenance category  $i$ .

Failures are binary events: either a failure occurs at a certain time or it does not. In Dinwoodie et al. (2015) the failure rates are given as the expected number,  $x_i$ , of failures per turbine per year for each maintenance category  $i$ . Failure rates of wind turbines can be expected to follow a bathtub curve over the life time of a wind farm (Hill et al., 2008). However, the rates can be considered (roughly) as constant if we disregard the infancy and end-of-life periods of a wind farm. Thus, within a planning horizon of one year, representing a normal operating year for a wind farm, it is reasonable to assume that failures are uniformly distributed, and that the expected number of failures per time period and turbine is the same for all time periods and equal to  $p_i = x_i/N$ , where  $N$  is the number of time periods. Based on this, the probability of exactly  $X$  failures in any given time period,  $P(\tilde{F}_{iq} = X)$ , is distributed according to a binomial distribution, and the value of  $F_{iqs}$  for each maintenance category, time period, and scenario is determined using the inverse sampling method on the cumulative density function of this distribution.

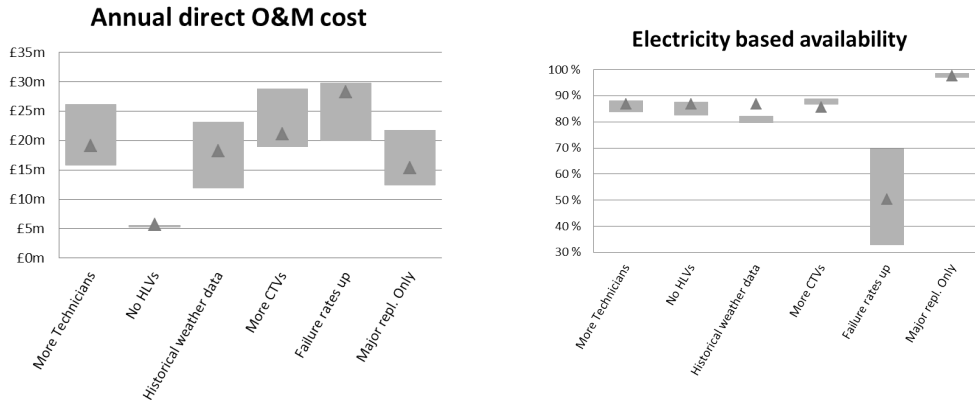


Figure 3: Comparison of the SP model with the simulation models presented in Dinwoodie et al. (2015) when it comes to average annual direct O&M cost and electricity based availability. The triangle represents the value obtained by the SP model, while the grey area represents the range of values obtained by the simulation models.

#### 4.3. Verification of the solution approach

To verify that our heuristic solution approach to the second stage problem gives a reasonable estimate of the yearly operating costs of a given fleet, we have tested it on some of the reference cases for operation and maintenance at offshore wind farms suggested by Dinwoodie et al. (2015). In their paper, four different simulation models are applied on a set of test cases based on a reference wind farm. Due to the nature of simulation models, they are able to model the costs of using one specific fleet in much greater detail than is possible by a tractable optimization model. However, we can use their results to verify that the cost estimates produced by the second stage of our model are reasonable, and thus sends the correct signals back to the first stage problem.

We have chosen to use the following cases: *More CTVs*, *More Technicians*, *Failure rates up*, *No HLVs*, *Historical weather data*, and *Major replacements only*. The reason why we have not tested the *Base case* from Dinwoodie et al. (2015) is that we have tested the model on historical weather data, which makes the case named *Historical weather data* a better comparison, since it removes one source of discrepancy: the weather generation procedures in the simulation models. The tests have been conducted by fixing the first stage decisions to correspond with the fleet used in Dinwoodie et al. (2015), and then solving the second stage model with 200 scenarios.

The comparison of our model to the results presented by Dinwoodie et al. (2015) related to direct O&M costs and electricity based availability can be seen in Figure 3. For all test cases the results are well within the range of results obtained by the four simulation models tested in Dinwoodie et al. (2015) when it comes to total O&M costs, while we are a off by a few percent in two of the cases when comparing the electricity based availability. These results are likely related to the weather resolution used in the scenarios, which is of a lower granularity than the simulation models, and the worst weather state within each time interval is used. Hence, our SP model slightly overestimates the downtime costs.

The most valuable comparison with the simulation models from Dinwoodie et al. (2015) is the comparison of total annual costs (including downtime costs) since it is this value that the mathematical model minimizes. This comparison is presented in Figure 4. As can be seen in the figure, the results are well within the range provided by the simulation models, and also in the lower end of the range for all cases except for case *Failure rates up*. For this case, the reason is that the simulation models are unable to execute all the preventive maintenance tasks within a year, in contrast to our model. As a consequence the total cost of repairs increases, even though the other costs are comparable.

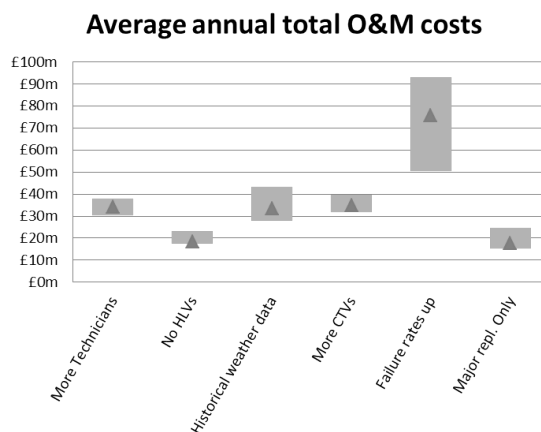


Figure 4: Comparison of the SP model and the simulation models presented by Dinwoodie et al. (2015) when it comes to average annual total cost including downtime costs. The triangle represents the value obtained by the SP model, while the grey area represents the range of values obtained by the simulation models.

#### 4.4. Testing the stochastic programming model

Having confirmed that the SP model gives reliable output for a fixed fleet of vessels, we next need to verify that the results of the SP model are not just dependent on the generated scenarios, but on the actual uncertainty of the problem. To achieve this we have adopted the approach from King & Wallace (2012) which suggests two procedures: In-sample and out-of-sample stability tests. These tests also provide good indications on the number of scenarios needed to obtain stable results from the model. All the following tests are based on the *No HLVs* test case from Dinwoodie et al. (2015), with the additional vessels from Table A.4.

The in-sample stability tests show to what degree the optimal objective value of the mathematical model varies for different sets of scenarios. Let  $\hat{x}_i$  be the optimal solution obtained with scenario-tree  $\Gamma_i$  and  $f(\hat{x}_i, \Gamma_i)$  the corresponding objective value. The in-sample stability test can then be seen as verifying that

$$f(\hat{x}_i, \Gamma_i) \approx f(\hat{x}_j, \Gamma_j)$$

holds for all pairs of test instances  $i$  and  $j$ .

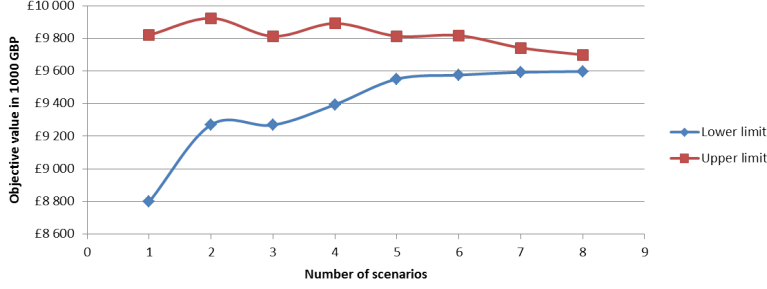


Figure 5: Figure illustrates how the 95 % confidence interval for the in-sample stability tests changes as a function of the number of scenarios in the scenario-tree.

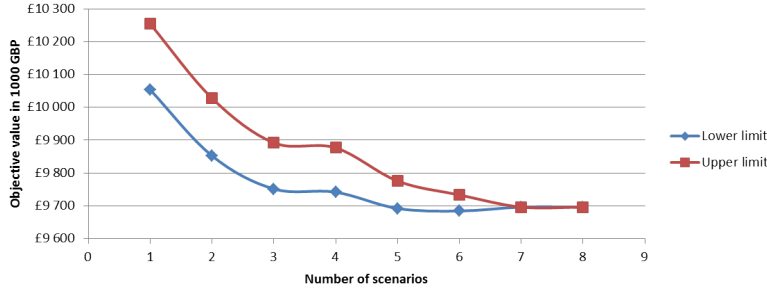


Figure 6: Figure illustrates how the 95 % confidence interval for the out-of-sample stability tests changes as a function of the number of scenarios in the scenario-tree.

To test the in-sample stability, we created 50 scenario-trees for each scenario-tree size from 1-8, and solved all the resulting instances. For each scenario-tree size, we then calculated the average objective function value, and the standard deviation. Based on these values we created 95 % confidence intervals for the optimal objective value for each scenario-tree size. A narrow interval indicates good in-sample stability.

Figure 5 shows the upper and lower limits of the 95 % confidence interval for the optimal objective value as the number of scenarios increases. We observe that the average objective function value increases with an increase in the number of scenarios. The reason for this is that the solutions become less tailored to the specific realisation of the uncertain parameters of a given scenario. Further, the width of the confidence interval narrows with the increase in number of scenarios, deviating by less than 0.5 % from the average value in the 8 scenario case. This indicates that the results become more stable and are less dependent on the individual scenario.

Out-of-sample stability is checked by taking the optimal solutions from tests with different scenario-trees and then calculate the solutions' true objective value. Let  $\mathcal{E}$  be the true distribution of the uncertainty in the model. The out-of-sample stability can then be expressed as verifying that

$$f(\hat{x}_i, \mathcal{E}) \approx f(\hat{x}_j, \mathcal{E})$$

holds for all pairs of test instances  $i$  and  $j$ .

Since  $\mathcal{E}$  in this problem is a continuous distribution we approximate it by means of 200 i.i.d samples (scenarios) from the underlying distribution. Thus, for each solution  $\hat{x}_i$  from the in-sample tests, we solve each of the 200 one-scenario problems fixing the first-stage component of the solution to the first-stage component of  $\hat{x}_i$ . In addition, we have for these tests re-introduced the integral requirements on the  $\lambda$ -variables. Thus, we also get to test the impact of the relaxation in the stochastic model.

Figure 6 presents the results from the out-of-sample stability tests. We have, as for the in-sample tests, calculated the average and the standard deviation of  $f(\hat{x}_i, \mathcal{E})$  over all test instances  $i$  with the same number of scenarios, and presents the graph showing a 95 % confidence interval. We observe a decrease in the average objective value when the number of scenarios increases. This is expected since the stochastic model will find fewer solutions that are tailored to a specific scenario's realisation of uncertain parameters. Also, there is a decrease in the width of the confidence interval, and once we pass 7 scenarios, all scenario-trees give the same first stage solution to the problem, i.e. the same vessel fleet.

The objective of the in-sample and out-of-sample stability tests was to check that the solutions produced by our mathematical model come from the behaviour of the model itself, and not from the scenario generation. The tests show that once the number of scenarios exceeds 5 we get relatively stable results in terms of low variance in the objective values. Further, we see from the out-of-sample tests that once we exceed 7 scenarios we get the exact same out-of-sample objective value for all solutions. The out-of-sample stability tests converge to approximately 9.7 million GBP, while the in-sample stability tests converge to approximately 9.65 million GBP. This indicates that we lose little in terms of estimating the correct cost and fleet, by relaxing the  $\lambda$ -variables in the stochastic model. From the out-of-sample stability test we may also quantify the value of having a stochastic model, compared to using a deterministic model with one (randomly generated) realisation of each stochastic variable. The average error of using one scenario is almost 5 %, while the maximum error observed in the dataset is almost 14 %. We may thus conclude that there is significant value in applying stochastic programming to this problem.

To ensure that the presented model can be used to give decision support to planners, we must also consider the computing time. For a stochastic program, the computing time often varies with the size of the scenario-tree. We have therefore calculated the 95 % confidence interval for the computing time for each set of tests having the same number of scenarios. The results are presented in Figure 7. When comparing the results of the in-sample and out-of-sample tests with the computing times, it seems that 5 scenarios give a good balance between solution quality and computational time. Hence, we use this number of scenarios for the remainder of our tests.

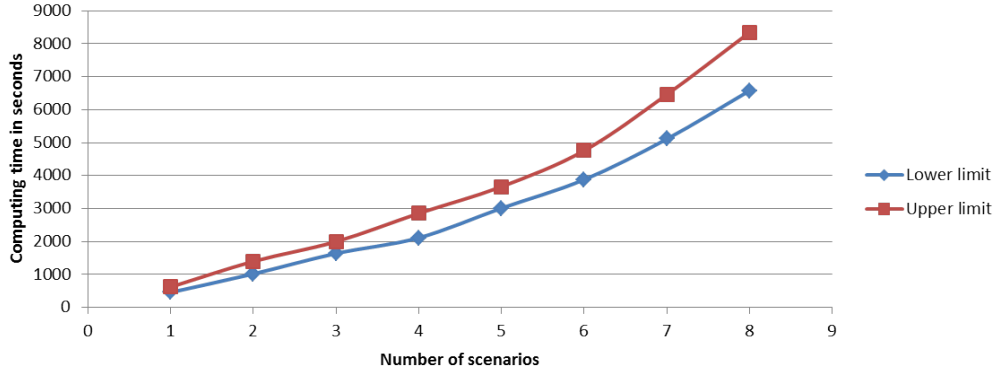


Figure 7: Figure illustrates how the 95 % confidence interval for computing time changes as a function of the number of scenarios in the scenario-tree.

#### 4.5. Managerial use of the model

To further test the applicability of the proposed method, and to exemplify the many applications such a model may have for decision makers, we have created a set of test cases. The summary of the results can be found in Table 1. For each of the test cases we give the energy based availability, the estimated total O&M cost, as well as a split of these costs into charter costs, downtime costs and spare part costs. Finally, we give the optimal fleet of vessels suggested by the model. The solution to the base case test instance is to use a fleet of 2 SES, at an approximate cost of 9.7 M GBP pr. year, giving the wind farm an energy based availability of 95 %.

##### 4.5.1. Updating an existing vessel fleet

For the reference case with the additional vessel types in Table A.4, the optimal vessel fleet is to long-term charter two surface effect ships (SES). We consider the case where the wind farm operator is committed to long-term charter two CTVs. In this case, the operator may wish to know whether it is profitable to complement the existing fleet of two CTVs by chartering one or more additional vessels. Hence, we solve the base case with additional vessel types given an existing fleet of two CTVs.

The optimal solution for this case is to charter in an additional SES, giving a fleet of two CTV and one SES. Although this fleet consists of 3 vessels, the downtime costs actually increase significantly compared with the base case. This can be explained by the CTVs having significantly lower wave height limits. Hence, there are fewer time periods where vessels can support maintenance tasks and the repair time will be prolonged as vessels need to wait for an appropriate weather window. Finally, we notice that the total O&M costs are actually lower for this case compared with the base case. This can be explained by the number of failures in the scenarios generated for this case is lower, leading to less maintenance tasks in total (can be verified by seeing that the spare part costs are lower). Thus, for this specific scenario set, a fleet of 2 SES would (probably) have resulted in even lower total O&M costs.

#### 4.5.2. *Planning a maintenance campaign*

Assume a wind farm operator with a fleet of two SES is planning a maintenance campaign one summer in addition to the regular yearly maintenance. The maintenance campaign consists of one maintenance task per turbine that requires 4 technicians for 50 hours, and all tasks should be executed between May and August the coming year. To facilitate the maintenance campaign, the operator may want to short-term charter some additional CTVs. We assume that the short-term charter rate for a CTV is 2000 GBP per day (compared to a long-term charter rate of 1750 GBP per day).

The optimal solution is to short-term charter two CTVs for one month (or one CTV for two months). It should be noted that the two SES in the permanent fleet are also used to support the maintenance tasks in the campaign. We see that the maintenance campaign negatively influences the availability of the wind farm, both because the wind turbines are stopped while the preventive maintenance tasks are performed, and because the total number of maintenance tasks means that some corrective maintenance tasks may be delayed.

#### 4.5.3. *Distance to shore*

How does increasing the distance to shore (i.e. the nearest onshore base) influence the optimal vessel fleet? We have created two new instances where the distance to the onshore base is 100 and 150 km, respectively.

Increasing the distance to shore to 100 km does not affect the optimal vessel fleet, however, the downtime costs increase significantly since longer travel times back and forth between the base and the wind farm means that there is less time available to perform maintenance during each shift. When extending the distance to 150 km, it is no longer possible to perform maintenance effectively from shore, and the model prefers to use a small accommodation vessel which stays at the wind farm for several time periods.

#### 4.5.4. *Determining long-term charter cost for a vessel*

The ship designer that has developed the *Small Accommodation Vessel* (SAV) may want to know at what long-term charter rate the SAV is competitive with the other vessel fleets for the wind farm in question. We can determine this by solving the model to optimality without the SAV, and then solve the model with the SAV fixed in the fleet at a charter rate of 0. We may then look at the difference between the objective values, to get an upper limit on when the SAV is a competitive alternative for this wind farm.

From the base case we see that the optimal solution without the SAV is  $\approx 9.70$  million GBP, while the optimal solution with the SAV with time charter rate 0 is  $\approx 6.51$  million GBP. Thus, we may conclude that the SAV should be priced in the region of 3.2 million GBP per year, or 8700 GBP per day to be a competitive alternative. This is significantly lower than the 12500 GBP per day we have used in the previous tests.



Table 1: Detailed results from the tests presented to demonstrate managerial use of the model.

	Base case	2 CTV in fleet	Campaign	100 km to shore	150 km to shore	SAV cost test
Availability - energy based	95.00 %	94.60 %	94.00 %	93.81 %	94.04 %	94.54 %
O&M cost (in GBP)	9 696 380	9 569 655	11 342 194	11 000 293	11 696 490	6 513 477
Downtime cost (in GBP)	5 400 635	5 830 955	6 168 244	6 724 093	6 464 480	5 835 682
Charter cost (in GBP)	3 650 000	3 102 500	4 528 750	3 650 000	4 562 500	0
Spare part costs (in GBP)	645 745	636 200	645 200	626 200	669 510	677 795
Optimal Fleet	2 SES	2 CTV + 1 SES	2 SES + 1 CTV for 2 months	2 SES	SAV	SAV

## 5. Conclusions

In this paper we have proposed a two-stage stochastic programming model for the vessel fleet size and mix problem to support O&M at offshore wind farms. This is a fairly new and relevant industry problem, where it is essential to find good or optimal solutions that will help reduce the cost of energy from offshore wind farms. To determine the optimal vessel fleet size and mix we have developed a new two-stage stochastic programming model that considers uncertainty in weather conditions and the occurrence of corrective O&M tasks. The model is based on an ad-hoc Dantzig-Wolfe decomposition, which differs from standard decomposition approaches for stochastic programming models by leaving parts of the second stage problem in the master problem. To solve the model for realistic instances a matheuristic is developed which is based on generating a subset of all possible columns.

Through a computational study we have shown that the presented model and solution approach give reasonable estimates on the electricity based availability, and the operating costs of a given vessel fleet, by comparing our approach to the results of several life-cycle cost estimating simulation models from the literature. Further, testing the in-sample and out-of-sample stability of the model provides stable output on a reference case from the literature with as little as 5 to 6 scenarios. This means that the method proposed is computationally tractable even though the number of variables in each scenario is large. Finally, we have shown how the model can provide decision support to different actors in the offshore wind business: for a wind farm operator it can e.g. be used to determine an optimal fleet and determine which vessels to short-term charter when planning a maintenance campaign, and for vessel developers the model can e.g. give insight into which time-charter rates their vessels need to be a competitive alternative for different offshore wind farms.

A recent trend in the offshore wind industry is that operators are looking to move from using a preventive maintenance strategy to a condition based maintenance strategy. Ideally, this would lead to the necessary maintenance tasks being performed shortly before the turbine fails, thus significantly reducing the downtime cost of the offshore wind farm. Thus far the lack of reliable condition monitoring systems have prevented condition based maintenance from being widely used. However, a lot of research is currently ongoing to improve these systems, and it is likely that this strategy will be commonly used in the future. A natural next step of this research is thus to

incorporate condition based maintenance tasks into the model.

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## Appendix A. Input data for the computational study

Table A.1: Overview of the failures from the reference case presented by Dinwoodie et al. (2015).

FAILURE INPUT	Manual reset	Minor repair	Medium repair	Major repair	Major replacement	Annual service
Repair time	3 hours	7.5 hours	22 hours	26 hours	52 hours	60 hours
# Technicians	2	2	3	4	5	3
Vessel type	CTV	CTV	CTV	FSV	HLV	CTV
Failure rate	7.5	3	0.275	0.04	0.08	1
Repair cost (GBP)	0	1000	18 500	73 500	334 500	18 500

Table A.2: Overview of the vessel types from the reference case presented by Dinwoodie et al. (2015).

VESSEL INPUT	Crew Transfer Vessel (CTV)	Field Support Vessel (FSV)	Heavy-Lift Vessel (HLV)
Number of vessels	3	1	1
Governing weather criteria	Wave	Wave	Wave / Wind
Weather criteria	1.5 m	1.5 m	2.0 m / 10.0 m/s
Speed of vessel	20 knots	12 knots	11 knots
Technician capacity	12	60	100
Day rate (GBP)	1750	9500	150 000
Maximum offshore time	1 shift	4 weeks	No limit

Table A.3: Overview of the wind farm data from the reference case presented by Dinwoodie et al. (2015).

Other test data	Value
Number of turbines	80
Distance maintenance base to wind farm	50 km
Wind and wave weather data	FINO [16]
Technician cost	80 000 GBP/year
Number of technicians available	20
Working shift	12 hours
Number of daily shifts	1
Price of electricity	90 GBP/MWh
Wind turbine power curve	Based on V90 power curve
Cut-in and cut-out speeds	3 m/s, 25 m/s

Table A.4: Overview of the vessel types used by Sperstad et al. (2017).

Vessel Input	Surface Effect Ship (SES)	Small Accommodation Vessel	Mini mother vessel	Daughter vessel
Governing weather criteria	Wave	Wave	Wave	Wave
Weather criteria	2.0 m	2.0 m	2.5 m	1.2 m
Speed of vessel	35 knots	20 knots	14 knots	16 knots
Technicians available pr. shift	12	12	16	6
Number of shifts pr. day	1	2	2	2
Day rate	5000	12500	25000	0
Maximum offshore time	1 shift	2 weeks	2 weeks	1 shift

## Appendix B. Notation used in the mathematical models

### Set definitions

$\mathcal{A}$	Set of arcs $((i, m), (j, n))$ between nodes $(i, m), (j, n) \in \mathcal{N}$ in the problem defining graph
$\mathcal{K}$	Set of bases
$\mathcal{M}$	Set of all categories of maintenance tasks
$\mathcal{M}^C$	Set of all categories of corrective maintenance tasks
$\mathcal{M}^L$	Set of categories of corrective maintenance tasks that must be performed by a vessel with lifting capabilities
$\mathcal{M}^P$	Set of all categories of preventive maintenance tasks
$\mathcal{M}^T$	Set of categories of corrective maintenance tasks that only requires technicians present
$\mathcal{N}$	Set of nodes $(i, m)$ in the problem defining graph
$\mathcal{P}$	Set of time period in the planning horizon
$\mathcal{V}$	Set of available vessel types
$\mathcal{V}_k$	Set of available vessel types that can operate from base $k$ , $k \in \mathcal{K}$ , $\mathcal{V}_k \subseteq \mathcal{V}$
$\mathcal{V}^L$	Set of vessel types with lifting capabilities $\mathcal{V}^L \subseteq \mathcal{V}$
$\mathcal{T}$	Set of sets of time periods for short-term charter of vessels, $t_1, t_2 \in \mathcal{T}$ , $t_1, t_2 \subset \mathcal{P}$ , $t_1 \cap t_2 = \emptyset$
$\mathcal{W}_{kv}$	Set of all possible maintenance patterns that may be executed by a vessel of type $v \in \mathcal{V}$ operating from base $k \in \mathcal{K}$
$\mathcal{S}$	Set of scenarios

### Deterministic Parameters

$A_i$	Number of preventive maintenance activities of category $i \in \mathcal{M}^C$ that needs to be supported during the planning horizon
$A_{iw}$	Number of maintenance tasks of category $i \in \mathcal{M}^T$ that are executed in maintenance pattern $w \in \mathcal{W}_{kv}$ for a given combination of $k \in \mathcal{K}$ and $v \in \mathcal{V}$
$B_{k_1 k_2}$	Parameter equal to 1 if bases $k_1 \in \mathcal{K}$ and $k_2 \in \mathcal{K}$ can both be used at the same time, 0 otherwise
$C_w$	The cost of performing maintenance pattern $w$
$C_k^B$	Fixed yearly cost of operating base $k \in \mathcal{K}$
$C_{ps}^D$	The realisation of $\tilde{C}_p^D$ in scenario $s \in \mathcal{S}$
$C^F$	The fuel cost
$C_v^{LT}$	Charter cost for a long-term charter of vessel type $v \in \mathcal{V}$
$C_{vt}^{ST}$	Time charter cost for vessel type $v \in \mathcal{V}$ for the time charter period $t \in \mathcal{T}$
$C_i^P$	Penalty cost of not completing a maintenance task of category $i \in \mathcal{M}$
$E_k$	Equal to 1 if base $k$ must be used, 0 otherwise, $k \in \mathcal{K}$
$E_{kv}$	Number of vessel type $v \in \mathcal{V}$ at base $k \in \mathcal{K}$ that must be used



$F_v$	The fuel consumption per time unit of vessel type $v \in \mathcal{V}$
$F_{iqs}$	The realisation of $\tilde{F}_{iq}$ in scenario $s \in \mathcal{S}$
$L_i$	The number of technicians needed to perform maintenance task $i \in \mathcal{M}^T \cup \mathcal{M}^P$
$M_v$	The maximum number of technicians onboard a vessel of type $v \in \mathcal{V}$
$N$	equal to $ \mathcal{M}^T $
$p_s$	Probability of scenario $s \in \mathcal{S}$
$Q_{kv}^{MX}$	Maximum number of vessel type $v \in \mathcal{V}$ that can be stationed at base $k \in \mathcal{K}$
$Q_{vt}^{MX}$	Maximum number of vessel type $v \in \mathcal{V}$ available for time charter in charter period $t \in \mathcal{T}$
$T_w$	The total duration of executing maintenance pattern $w \in \mathcal{W}_{kv}$ for a given vessel type $v \in \mathcal{V}$ stationed at base $k \in \mathcal{K}$
$T_i^M$	The time it takes to perform maintenance task $i \in \mathcal{M}$
$T_{vi}$	The average travel time associated with using vessel type $v \in \mathcal{V}$ when performing maintenance task $i \in \mathcal{M}$ , including the time it takes to transfer technicians and equipment to the turbine
$T_{kv}^{max}$	The maximum time vessel $v \in \mathcal{V}$ stationed at base $k \in \mathcal{K}$ may operate in any time period.
$T_{vps}^{max}$	The realisation of $\tilde{T}_{vp}^{max}$ in scenario $s \in \mathcal{S}$
$U_{iv}$	An upper bound on the number of maintenance tasks of type $i \in \mathcal{M}^T \cup \mathcal{M}^P$ that can be performed by vessel $v \in \mathcal{V}$ in a single time period.
$P_s$	Probability of scenario $s \in \mathcal{S}$

#### Stochastic Parameters

$\tilde{C}_p^D$	Downtime cost of not producing electricity in time period $p \in \mathcal{P}$
$\tilde{C}_{qp}^D$	Downtime cost of supporting a corrective maintenance task in time period $p \in \mathcal{P}$ that occurred in time period $q \in \mathcal{P}$
$\tilde{F}_{iq}$	The number of maintenance tasks of category $i \in \mathcal{M}^C$ that occurs in time period $q \in \mathcal{P}$
$\tilde{T}_{vpi}^L$	The number of time periods required for vessel $v \in \mathcal{V}$ to perform a maintenance task of category $i \in \mathcal{M}^L$ if it is started in period $p \in \mathcal{P}$
$\tilde{T}_{vp}^{max}$	The maximum amount of time vessel type $v \in \mathcal{V}$ can operate in time period $p \in \mathcal{P}$

#### Variables

$\beta_k$	One if base $k \in \mathcal{K}$ is part of the solution
$\gamma_{kv}^{LT}$	The number of vessels of type $v \in \mathcal{V}$ stationed at base $k \in \mathcal{K}$ that long-term chartered
$\gamma_{kvt}^{ST}$	The number of vessels of type $v \in \mathcal{V}$ stationed at base $k \in \mathcal{K}$ that is short-term chartered in the set of time periods $t \in \mathcal{T}$
$\lambda_{kvwp}$	The number of maintenance patterns $w \in \mathcal{W}_{kv}$ that is performed by vessel type $v \in \mathcal{V}$ operating from base $k \in \mathcal{K}$ in time period $p \in \mathcal{P}$
$l_{im}$	The number of technicians onboard the vessel when leaving node $(i, m) \in \mathcal{N}$

$t_{im}$	The time node $(i, m) \in \mathcal{N}$ is visited
$u_{kvp_i}$	The number of maintenance tasks of category $i \in \mathcal{M}^L$ is performed by vessel type $v \in \mathcal{V}$ operating from base $k \in \mathcal{K}$ in time period $p \in \mathcal{P}$
$x_{imjn}$	Equal to one if the vessel considered traverses arc $((i, m), (j, n)) \in \mathcal{A}$ , zero otherwise
$y_{qpi}$	The number of maintenance tasks of category $i \in \mathcal{M}$ that is performed in period $p \in \mathcal{P}$ that supports a failure that happened in period $q \in \mathcal{P}$
$z_i$	The number of maintenance tasks of category $i \in \mathcal{M}$ that have not been supported during the planning horizon.