Risk Control in Maritime Shipping Investments

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Abstract

In this paper we extend the state-of-the-art stochastic programming models for the Maritime Fleet Renewal Problem (MFRP) to explicitly limit the risk of insolvency due to negative cash flows when making maritime shipping investments. This is achieved by modeling the payment of ships in a number of periodical installments rather than in a lump sum paid upfront, representing more closely the actual cash flows for a shipping company. Based on this, we propose two alternative risk control measures, where the first imposes that the cash flow in each time period is always higher than a desired threshold, while the second limits the Conditional Value-at-Risk. We test the two models on realistic test instances based on data from a shipping company. The computational study demonstrates how the two models can be used to assess the trade-offs between risk of insolvency and expected profits in the MFRP.

Keywords: Maritime Transportation, Maritime Fleet Renewal; Risk Control, Stochastic Programming, Conditional Value-at-Risk

1 Introduction

Ocean shipping companies enable trading between countries and continents, and are thus the backbone of the modern globalized economy. For such companies, decisions regarding the size and composition of the fleet are decisive not only for their competitiveness but also for their survival in an extremely competitive market. In fact, the risk of being insolvent in the shipping industry is not rare. One example is provided by the bankruptcy of Hanjin Shipping, the sixth largest container shipping company, in the fall 2016, see, e.g., BBC News (2017). In addition, according to The Economist (2016), a number of other shipping companies are in a vulnerable

position. Overcapacity in the industry is certainly one of the main reasons behind such vulnerability. UNCTAD (2016) reports a 3.5% growth in the world fleet from 2015 to 2016, despite an only 2.1% demand growth. As a consequence "*in 2015, most shipping segments, except for tankers, suffered historic low levels of freight rates and weak earnings*"(UNCTAD, 2016).

Furthermore, reducing the fleet by scrapping ships is not always a viable option due to low steel prices, as in current times. According to The Economist (2016) overcapacity might actually be triggered by big players which, by increasing capacity, drop freight rates to unprofitable levels for smaller players and in turn force them out of business. Therefore, analytic support for decisions regarding shipping investments must necessarily envisage also the possibility of market scenarios in which freight rates, demands, and scrapping rates fall to unprofitable levels, and suggest decisions which protect the company from positions of insolvency.

The problem of deciding the size and composition of a fleet of ships has, for many years received little attention by the Operations Research (OR) community. Pantuso et al. (2014) report only 37 scientific contributions produced in more than fifty years. However, this trend has recently been inverted, with a prolific research effort during the past five years. Particularly, the literature puts a special emphasis on the treatment of the uncertainty which characterizes shipping markets.

Alvarez et al. (2011) propose a robust optimization model with the scope of ensuring fleet renewal plans which are feasible despite random variation in the purchase and selling prices of ships. Bakkehaug et al. (2014) propose a multistage stochastic program in which a random variable, modeling the "status of the shipping market", controls a number of associated random parameters such as demand, ship prices, and charter rates. Pantuso et al. (2016) also present a multistage stochastic program for the *Maritime Fleet Renewal Problem* (MFRP) in which a number of market parameters (such as steel prices, demands, and charter rates) are not perfectly correlated. The authors show that explicitly facing uncertainty can significantly improve fleet renewal plans. A solution method for large-scale instances of the problem is offered by Pantuso et al. (2015), while Pantuso et al. (2017) show that, information related to expected values and range of variation of the demand plays an important role in fleet renewal plans.

Patricksson et al. (2015) extend the MFRP in order to deal with the limitation imposed in certain emission control areas. Particularly, among other possible actions, the authors include the possibility to upgrade existing ships to standards which would allow them to sail within emission control areas. Arslan and Papageorgiou (2017) consider the MFRP from the point

of view of an industrial bulk shipping company which needs to decide the number, the size and the duration of time charters. The authors also propose a multistage stochastic program which is solved using a rolling horizon heuristic. Finally, Mørch et al. (2017) revisit the mathematical model by Pantuso et al. (2016) proposing a model which maximizes the rate of return on the investments made. The authors show that such a model allows to match more closely the investors' preferences. Earlier methods and additional discussion on the MFRP can be found in the literature survey provided by Pantuso et al. (2014).

The above mentioned research assumes risk neutral decision makers which maximize expected profits/returns (or minimize expected costs). Therefore, the models proposed are not designed to hedge against particularly negative market configurations and, e.g., limit the risk of insolvency. In fact, while they produce solutions which are the best on average, they do not exclude that such solutions, in certain scenarios, might produce extremely negative cash flows. Thus, the available models do not necessarily protect the company in tough periods.

In this paper we take the perspective of a risk averse decision maker and study the problem of limiting the risk of insolvency when making shipping investment decisions. Particularly, we extend a state-of-the-art multistage stochastic programming formulation in order for it to explicitly limit excessively negative cash flows which might drive the company into a position of insolvency. We achieve this by proposing a number of modifications to the available model.

First, we take into account that the payment of ships is typically made in a number of installments. This is in contrast with the available literature which assumes that ships are fully paid in one lump sum (see for example Alvarez et al. (2011), Pantuso et al. (2015, 2016)). Stopford (2009) explains that the payment of new ships is usually made in at least three installments following corresponding milestones in the construction process. However, when ships are paid by debt, the ship is typically fully paid in five to ten years. Thus, by modeling periodical installments we are able to replicate more closely the cash flows of the company, and thus enforce control measures.

Second, we limit the negative magnitude of cash flows by means of two alternative risk control measures. The first type of measure imposes that the cash flow is higher than a desired (possibly negative) threshold in all possible scenarios (i.e., with probability one). This deterministic measure enables the company to ensure that cash flows always are higher than a certain company-specific safety threshold to avoid insolvency. The second type of risk control measure limits instead the Conditional Value-at-Risk (CVaR) of the negative cash flows, i.e., the expected negative cash flows in the worst-case tail of the cash flows distribution. With such risk control measure, the company is able to impose, for example, that the expected cash flow in the 5%-probability worst-case scenarios, is higher than $ -50 million.

Enforcing such controls on the negative cash flows might however have a negative impact on the expected profits by limiting the investment options available to the decision maker. Therefore, by considering a risk neutral decision maker as a benchmark, we study the trade-off between different degrees of risk awareness and expected profits.

The contributions of this paper is thus a novel multistage stochastic program which, with respect to the available literature, includes

- a closer and more realistic representation of the payment of ships in instalments rather than in a lump sum,
- a risk control measure which deterministically limits negative cash flows, and
- a risk control measure which limits the expected worst-case cash flow.

The novel representation of the payment of investments also requires changing the objective function, compared to previous models. In addition, for the resulting multistage stochastic program, we illustrate a node formulation which enables the solution of the problem through commercial solvers. Finally, we propose a computational study where the new model is tested on instances based on data from a real shipping company. In the computational study we show the effect of risk control measures on profits and derive consequent managerial and practical insights for shipping companies.

The remainder of this paper is organized as follows. In Section 2 we describe the MFRP with cash flow control in more detail, while a mathematical model for the problem is proposed in Section 3. In Section 4 we report from our computational study, and finally we draw conclusions in Section 5.

2 The Maritime Fleet Renewal Problem with Cash Flow Risk Control

In this section we introduce the MFRP with cash flows risk control. The problem revisits and extends the profit-maximization version described by Mørch et al. (2017) and is consistent with most features included in Pantuso et al. (2015, 2016, 2017). We begin by providing a general description of the problem.

A shipping company is to decide how to modify the available fleet of ships by adding or removing ships. Ships can be ordered from a ship-builder or bought in the second-hand market. In the former case the delivery of the ship takes typically a number of years, depending on the order-book of the ship-builder. In the latter case the delivery time is significantly

shorter, and depends essentially on administration tasks and on the position of the ship. A shipping company can also sell ships in the second-hand market, or scrap (demolish) them receiving a remuneration for the steel of the ship.

Ships are paid for in different ways. The payment to the ship-builder is typically delivered in three installments, the first at the placement of the order, the second at the lay of the keel, the last at the delivery of the ship (see Stopford, 2009). However, the actual cash outflow from the shipping company depends on whether the ship is financed by equity or debt. Typically, the cost of the ship (plus interests or dividends) is actually paid back in a number of installments for a period of up to ten years (see Stopford, 2009).

The ownership and operations of a ship generate fixed and variable operating expenses for the shipping company. Fixed operating expenses (typically referred to as OPEX) cover those costs which are not determined by the activities of the ship, such as insurance, administration costs, crew salaries, and maintenance. Fixed operating expenses can be lowered by laying up ships, i. e., stopping them at ports, due to, e.g., lower insurance fees and reduced crew. Variable operating expenses are generated by the sailing of the ship and can essentially be restricted to bunker costs, and port and canal fees.

Additional ships, for short-term needs, are typically obtained by time-charters. Time-charters give the charterer the control of a ship and its crew for a specified period of time (weeks to months). The charterer pays a (per day or per week) fee and all variable operating costs (e.g., fuel and port fees), while the charteree maintains the ownership of the ship and bears all capital costs and fixed operating expenses. Similarly, the shipping company has the possibility to charter out own ships.

The types and number of ships to operate is essentially determined by sailing needs which in turn are generated by a transportation demand, possibly contractualized. However, different configurations of the sailing operation can be found. Lawrence (1972) distinguishes among three modes, namely *industrial*, *liner*, and *tramp* shipping. In industrial shipping a producer of goods owns and operates a fleet of ships used to deliver its production to customers. In liner shipping, similarly to a city bus, the company deploys the fleet on predefined *trades*, i. e., fixed routes with a pre-published schedule. Finally, in tramp shipping ships are assigned to customers' transportation calls, like taxis. In Section 3 we assume liner shipping operations, while a more thorough description of the other modes can be found in Christiansen et al. (2007). In any case, unfulfilled transportation demand is typically covered by *space-charters*, i.e., by transporting products by means of other shipping companies' ships, or by paying a penalty to customers. Both options are typically expensive.

Due to the long lifetimes of ships and lead times for the delivery of new buildings, fleet

renewal plans need to take into account a planning horizon of a number of years. Consequently, several elements of the problem are uncertain when decisions are made, such as demands, ship purchase and selling prices, charter rates, steel prices, and bunker prices. Thus, fleet renewal plans are made under uncertainty. Finally, such decisions are made periodically, e.g., every year.

In every period, a shipping company receives cash inflows generated by the remuneration of the transportation services provided, by chartering own ships to other companies, and by selling or demolishing own ships. Cash outflows are instead generated by the payment in installments of the ships purchased, by the payment of fixed and variable operating expenses, by the time-charters taken, and by the space-charters used to cover unfulfilled demand. Ensuring solvency corresponds to ensuring that the net cash flow is, in every period, within a company-specific safety margin.

The MFRP with cash flow control consists of determining how many ships of each type to add to, or remove from, the available fleet in order to maximize expected profit while limiting the risk of insolvency due to cash flows falling below a company-specific safety margin. While MFRP decisions are made periodically, the focus is on the decisions which must be made here-and-now, while taking into account possible future scenarios and corresponding decisions.

3 Mathematical Model

In this section the MFRP with risk control is modeled as a multistage stochastic integer program. The multistage and stochastic structure allows us to capture the interplay between periodic decisions conditional on the discovery of new information (i.e., realizations of uncertain parameters). Modeling assumptions are discussed in Section 3.1, while in Section 3.2 we introduce the basic profit maximization model with payment of ships in installments (but without any risk control measures). Further, we propose two alternative measures for controlling cash flows. The first, presented in Section 3.3, is a deterministic measure restricting the cash flows to remain higher than a company-specific safety margin for all possible scenarios. The second measure, presented in Section 3.4, controls the Conditional Value-at-Risk (CVaR), i. e., it limits the expected negative cash flows in the tail of the distribution.

3.1 Modeling Assumptions

We make the following assumptions.

A1 We assume a finite planning horizon consisting of a finite number of decision times, i.e., stages. This, in practice, corresponds to making fleet renewal decisions periodically as

it is often the case in shipping companies.

A2 We assume that the joint probability distributions of the random parameters are known. This implies that the company at least implements routines to collect market data and estimate empirical distributions. Particularly, we assume a discrete distribution in the form of a finite set of scenarios and the respective probabilities. If the estimated distribution is continuous it can be discretized using standard scenario generation techniques such as Høyland et al. (2003) or sampling techniques.

A3 We assume that ships are different from each other in technology (i.e., speed, capacity, cost structure) and age. Thus, a specific configuration of technology and age determines a *ship class*. Notice therefore that two ships with identical technology, but built in two different years, belong to two different ship types.

A4 We assume second-hand ships that are bought in one period are delivered at the beginning of the next planning period. Similarly, we assume ships scrapped and sold leave the fleet at the end before the beginning of the next planning period. We assume new buildings are delivered after a suitable number of periods (lead time) which depends on the order book at the shipbuilder.

A5 We assume time-charters can be issued for at most one time period at a time (i.e., fractions of a period and up to an entire period). Time charters longer than a period must thus be issued one period at a time. Similarly, ships can be laid up for at most one period at a time.

A6 We assume that the shipping company operates in the liner shipping business. The corresponding shipping operations are described in what follows.

Consistently with Pantuso et al. (2016) and Mørch et al. (2017), the company has to service a number of trades, i.e., sequences of ports which have to be visited according to a pre-published schedule. A trade consists of a number of origin ports and a number of destination ports. A ship services a trade when it visits all its ports, according to the specific schedule, picking up cargoes at origin ports and delivering cargoes at destination ports. Figure 1 shows an example trade which includes five origin and three destinations ports. Transportation demands (possibly for different products) are associated to each origin-destination pair.

We inherit the graph representation of the network of trades used in Pantuso et al. (2016). Nodes in the graph represent trades. Visiting a node corresponds to servicing the trade it represents. Each node is assigned a demand which is calculated as the sum of the demands between its port pairs. When a ship visits a node it transports an amount of cargo up to the

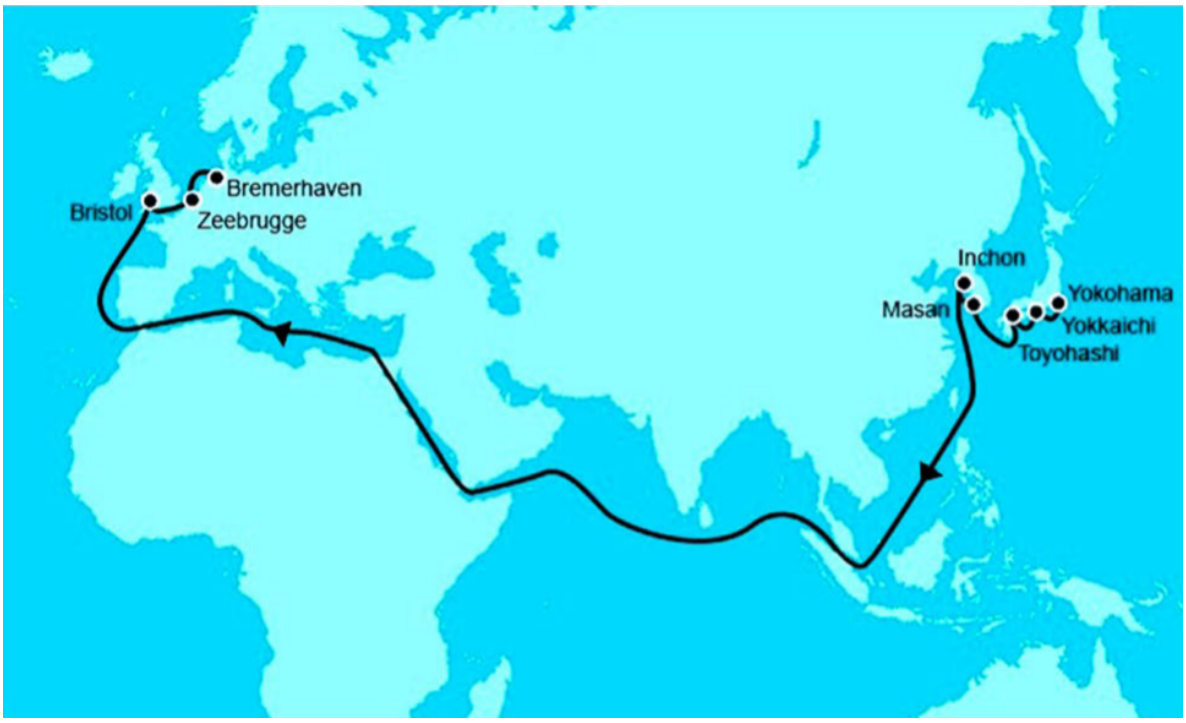


Figure 1: Example trade from Asia (with five origin ports) to Europe (with three destination ports).

capacity of the ship. Arcs represent ballast (i.e., empty) sailings between the last and the first port of the trades connected. As an example, in the graph depicted in Figure 2 the arc between trade $T1$ and trade $T2$ represents the ballast sailing between the last port in trade $T1$ and the first port in trade $T2$. It should be noted that the example Figure 2 includes four trades, and has no connection with the example showing one trade in Figure 1.

To perfectly assess the needed fleet capacity we would need to include detailed deployment and routing decisions on an operational level. However, this would result in an intractable model. To obtain a tractable model with a fair estimate of the capacity needed at the operational level, we adopt the concept of *loop* from Pantuso et al. (2016). A *loop* is a cyclic path in the graph, i. e., a path in the graph which begins and ends in the same node (trade). Ships are assigned to loops. A ship assigned to a loop services the trades in the loop in a given sequence, possibly with ballast sailings in between, and returns to the initial port of the first trade in the loop. The total length of a loop accounts for both the length of the ballast sailings and the length of the trades. The cardinality of a loop is equal to the number of trades it includes. Figure 2 shows two example loops, namely L_1, of cardinality three (it includes T_1, T_2, T_4), and L_2 of cardinality two (it includes T_2 and T_3). Including loops of higher cardinality corresponds to modeling the tactical deployment problem with a higher granularity. Based on the results from Pantuso et al. (2016), we have chosen in this paper to

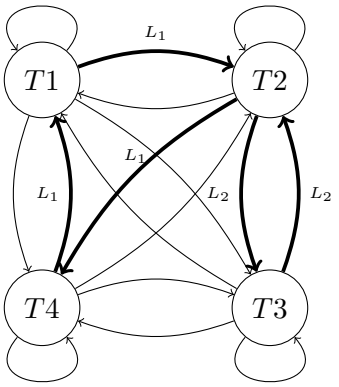


Figure 2: Graph representation of an example network with four trades and two loops.

include all loops with cardinality of one and two.

With respect to the sailing operations we make the following additional assumptions.

A7 We assume trades are either *contracted* or *optional*. Contracted trades are mandatory due to ongoing contracts which commit the company to sail from and to certain ports. Therefore, their demand must be fulfilled for the whole planning horizon. The company can instead decide to service each optional trade. However, once a company chooses to service an optional trade, that trade must be serviced for the remainder of the planning horizon.

A8 We assume space charters can be used only on contracted trades but not on optional trades. This corresponds to committing the company's own resources on the new sailing operations.

3.2 Basic Model: Profit Maximization without Cash Flow Control

In this section we first introduce the notation for the basic profit maximization model without cash flow control. Afterwards, we introduce and discuss the mathematical model. For the sake of legibility, all monetary quantities are to be considered appropriately discounted.

Sets	
$\mathcal{T}$	Set of time periods, indexed by t
$\mathcal{S}$	Set of scenarios, indexed by s
$\mathcal{K}$	Set of products, indexed by k
$\mathcal{V}_t$	Set of ship types existing in the market in period t, indexed by v (i.e. the ship types with age between zero and the retirement age)
$\mathcal{V}_t^N$	Set of potential new deliveries period t, i.e. the ship types with the age equal to zero in period t

$\mathcal{V}_t^{IN}$	The set of ship types for which the company pays instalments in period t
$\mathcal{N}_t$	Set of available trades in period t, indexed by i. $\mathcal{N}_t = \mathcal{N}_t^C \cup \mathcal{N}_t^O$
$\mathcal{N}_t^C \subseteq \mathcal{N}_t$	Set of contractual trades in period t, indexed by i
$\mathcal{N}_t^O \subseteq \mathcal{N}_t$	Set of optional trades in period t, indexed by i
$\mathcal{R}_t$	Set of loops available for sailing in period t, indexed by r
$\mathcal{R}_{vt} \subseteq \mathcal{R}_t$	Set of loops that can be sailed by ship type v in period t , indexed by r
$\mathcal{R}_{ivt} \subseteq \mathcal{R}_{vt}$	The set of loops servicing trade i that can be sailed in period t by a ship of type v, indexed by r

Parameters

$\bar{T}_v^L$	The lead time for the delivery of a ship of type v, i.e. the time between order placement and delivery
P_s	The probability for scenario s to take place, set to 1 divided by the number of scenarios
R_{vs}^{SV}	The sunset value of a ship of type v, in scenario s, i.e. the value of the ship at the end of the planning horizon
R_{its}^D	The revenue for transporting one unit of product on trade i, at period t and scenario s
R_{vts}^{SE}	The revenue for selling a ship of type v, in period t and scenario s
R_{vts}^{SC}	The scrapping value of ship of type v, in period t and scenario s
R_{vts}^{LU}	The lay-up savings for one period, for ship of type v, in period t and scenario s
R_{vts}^{CO}	The one-period charter-out revenue for ship of type v, in period t and scenario s
C_{vts}^{CI}	The charter-in cost for a ship of type v, in period t and scenario s
C_{vts}^{OP}	The operating cost for a ship of type v, in period t and scenario s
C_{vrts}^{TR}	The cost of performing loop r with a ship of type v, in period t and scenario s
C_{ikts}^{SP}	The cost of delivering one unit of product k on trade i by space charters, in period t, and scenario s
$\overline{CI}_{vts}$	The maximal number of charters in of a ship of type v, in period t and scenario s.
$\overline{CO}_{vts}$	The maximal number of charters out of a ship of type v, in period t and scenario s
$\overline{CI}_{ts}$	The maximal total number of charters in, in period t and scenario s
$\overline{CO}_{ts}$	The maximal total number of charters out, in period t and scenario s
$\overline{SE}_{vts}$	The maximal number of second-hand sales of ships of type v, in period t and scenario s
$\overline{SH}_{vts}$	The maximal number of second-hand purchases of ships of type v, in period t and scenario s
$\overline{SE}_{ts}$	The maximal total number of second-hand sales, in period t and scenario s
$\overline{SH}_{ts}$	The maximal total number of second-hand purchases, in period t and scenario s
Q_{vk}	The capacity of product k on a ship of type v
Q_v	The total capacity on a ship of type v
Z_{vr}	The time a ship of type v needs to perform a loop r
Z_v	The total time available in a period (e.g, days in a year) for a ship of type v
D_{ikts}	The demand of product k on trade i in period t and scenario s

F_{it}	The frequency requirement on trade i in period t, i.e. the number of times a trade has to be serviced during one period
Y_v^{IP}	The initial fleet of ships of type v, i.e., the number of ships available at the beginning of the planning horizon
Y_{vt}^{NB}	The number of ships of type v ordered in the past (i.e., in previous planning periods) and delivered at the beginning of period t
$C_{vt'ts}^{IN}$	The instalment paid in period t and scenario s for a ship of type v ordered in period t'
$C_{vt'ts}^{SH}$	The instalment paid in period t and scenario s, on a ship of type v purchased in the second hand market at time t'
C_{vt}^{IN}	The instalment paid in period t for a ship of type v already in the fleet at the beginning of the planning horizon. Note that this parameter is not stochastic, because the ship has already been bought and thus the instalments are already determined
M	Number of instalments
$\bar{T}$	The last time period in the planning horizon
Variables	
y_{vts}^{SC}	The number of ships of type v scrapped in period t and scenario s
y_{vts}^{SE}	The number of ships of type v sold in the second hand market , in period t and scenario s
$y_{vt's}^{SH}$	The number of ships of type v bought in the second hand market, in period t' and scenario s
$y_{vt's}^{NB}$	The number of new buildings ordered for a ship of type v, in period t' and scenario s
y_{vts}^{P}	The number of ships of type v in the fleet, in period t and scenario s
l_{vts}	The number of ships of type v put on lay-up, in period t and scenario s
h_{vts}^{I}	The number of ships of type v chartered in, in period t and scenario s
h_{vts}^{O}	The number of ships of type v chartered out, in period t and scenario s
x_{vrts}	The number of loops r performed by ships of type v, in period t and scenario s
n_{ikts}	The amount of cargo k delivered by space charters on trade i, in period t and scenario s
δ_{its}	Binary variable set to 1 if the company chooses to service trade i in period t and scenario s, 0 otherwise.

The profit maximization MFRP without cash flow control can thus be formulated as follows.

$$\max z = \sum_{s\in\mathcal{S}} P_s \left(\sum_{t\in\mathcal{T}\setminus\{0\}} \left(\sum_{i\in\mathcal{N}_t^O} \sum_{k\in\mathcal{K}} R_{its}^D D_{ikts} \delta_{its} \right. \right. \tag{1a}$$

$$\left. \left. + \sum_{i\in\mathcal{N}_t^C} \sum_{k\in\mathcal{K}} \left(R_{its}^D D_{ikts} - C_{ikts}^{SP} n_{ikts} \right) \right. \right. \tag{1b}$$

$$- \sum_{v \in \mathcal{V}_t} \Bigg(C^{OP}_{vts} y^P_{vts} + C^{CI}_{vts} h^I_{vts} - R^{CO}_{vts} h^O_{vts} \tag{1c}$$

$$+ \sum_{r \in \mathcal{R}_{vt}} C^{TR}_{vrts} x_{vrts} - R^{LU}_{vts} l_{vts} \Bigg) \Bigg) \tag{1d}$$

$$- \sum_{t \leq M-1} \sum_{v \in \mathcal{V}^{IN}_t} C^{IN}_{vt} Y^{IP}_v \tag{1e}$$

$$- \sum_{t \in \mathcal{T}} \sum_{t-M+1 \leq t' \leq t} \sum_{v \in \mathcal{V}^{IN}_t} (C^{IN}_{vt'ts} y^{NB}_{vt's} + C^{SH}_{vt'ts} y^{SH}_{vt's}) \tag{1f}$$

$$+ \sum_{v \in \mathcal{V}_{\bar{T}}} (R^{SV}_{vs} y^P_{v\bar{T}s} - \sum_{t' \in \mathcal{T}} \sum_{t=\bar{T}}^{t'+M} (C^{IN}_{vt'ts} y^{NB}_{vt's} + C^{SH}_{vt'ts} y^{SH}_{vt's})) \tag{1g}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{v \in \mathcal{V}_t} (R^{SC}_{vts} y^{SC}_{vts} + R^{SE}_{vts} y^{SE}_{vts}) \Bigg) . \tag{1h}$$

Objective function (1a)–(1h) represents the expected profit for the whole planning horizon. The term in (1a) represents the revenue obtained for fulfilling the demand on optional trades. The terms in (1b) represent the revenue from contracted trades minus the expenses for space charters. The terms in (1c) represent the fixed operating expenses, the expenses for time charters and the revenue for time chartering own ships to other companies. The terms in (1d) represent the sailing expenses minus the savings for laying-up ships. The term in (1e) represents the installments that have to be paid for ships purchased in the past (i.e., in previous, separated, decision problems). The terms in (1f) sum up the instalments for the payment of ships built or bought in the second-hand market. The terms in (1g) represent the sunset value of the fleet minus the sum of the instalments that have to be paid after the end of the planning period due to purchases and new buildings decided within the end of the planning horizon. Finally, the terms in (1h) represent the revenue from scrapping and selling own ships.

The problem is subject to the following constraints.

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_{vk} x_{vrts} + n_{ikts} \geq D_{ikts} \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^C_t, k \in \mathcal{K}, s \in \mathcal{S}, \tag{1i}$$

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_{vk} x_{vrts} \geq D_{ikts} \delta_{its} \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^O_t, k \in \mathcal{K}, s \in \mathcal{S}, \tag{1j}$$

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_v x_{vrts} + \sum_{k \in \mathcal{K}} n_{ikts} \geq \sum_{k \in \mathcal{K}} D_{ikts} \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^C_t, s \in \mathcal{S}, \tag{1k}$$

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_v x_{vrts} \geq \sum_{k \in \mathcal{K}} D_{ikts} \delta_{its} \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^O_t, s \in \mathcal{S}. \tag{1l}$$

Constraints (1i) and (1j) ensure the satisfaction of the demand for all products on contracted trades and on optional trades, respectively. Notice that demand must be satisfied for all periods except the initial. In fact, the fleet composition available in the initial period is the result of an earlier planning problem, and the corresponding chartering decisions are made in a separated tactical-level problem. These decisions do not influence the investment decisions the MFRP focuses on. Notice also that space charters can be used only on contracted trades but not on optional trades (see Assumption A8 in Section 3.1). Constraints (1k) and (1l) ensure that the total capacity is sufficient to cover the demand on contracted and optional trades, respectively. Notice that these constraints are also modeled for all periods except the initial.

$$\sum_{v\in\mathcal{V}_t}\sum_{r\in\mathcal{R}_{ivt}} x_{vrts} \geq F_{it} \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^C, s\in\mathcal{S}, \qquad \text{(1m)}$$

$$\sum_{v\in\mathcal{V}_t}\sum_{r\in\mathcal{R}_{ivt}} x_{vrts} \geq F_{it}\delta_{its}, \qquad t\in\mathcal{T}\setminus\{0\}, i\in\mathcal{N}_t^O, s\in\mathcal{S}, \qquad \text{(1n)}$$

$$\sum_{r\in\mathcal{R}_{vt}} Z_{vr}x_{vrts} \leq Z_v(y_{vts}^P + h_{vts}^I - h_{vts}^O - l_{vts}), \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \qquad \text{(1o)}$$

$$\delta_{its} \leq \delta_{i,t+1,s} \qquad t\in\mathcal{T}\setminus\{0,\bar{T}\}, i\in\mathcal{N}_t^O, s\in\mathcal{S}. \qquad \text{(1p)}$$

Constraints (1m) and (1n) enforce the service frequency requirements on the contracted and optional trades, respectively. Constraints (1o) ensure that the fleet (including time charters) has enough ships to cover the required sailing time. Constraints (1p) ensure that, when the company decides to service an optional trade, it is serviced for the rest of the planning horizon (see Assumption 7 in Section 3.1.

$$y_{v0s}^P = Y_v^{IP} \qquad v\in\mathcal{V}_0, s\in\mathcal{S}, \qquad \text{(1q)}$$

$$y_{vts}^P = Y_{vt}^{NB} \qquad t\in\mathcal{T}: t<\bar{T_v}^L, v\in\mathcal{V}_t^N, s\in\mathcal{S}, \qquad \text{(1r)}$$

$$y_{vts}^P = y_{v,t-1,s}^P - y_{v,t-1,s}^{SC} + y_{v,t-1,s}^{SH} - y_{v,t-1,s}^{SE} \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t\setminus\mathcal{V}_t^N, s\in\mathcal{S}, \qquad \text{(1s)}$$

$$y_{vts}^P = y_{v,t-\bar{T}_v^L,s}^{NB} \qquad t\in\mathcal{T}: t\geq\bar{T}_v^L, v\in\mathcal{V}_t^N, s\in\mathcal{S}, \qquad \text{(1t)}$$

$$y_{vts}^P \geq l_{vts} - h_{vts}^I + h_{vts}^O \qquad t\in\mathcal{T}\setminus\{0\}, v\in\mathcal{V}_t, s\in\mathcal{S}, \qquad \text{(1u)}$$

$$y_{vts}^P = y_{vts}^{SC} \qquad t\in\mathcal{T}\setminus\{\bar{T}\}, v\in\mathcal{V}_t\setminus\mathcal{V}_{t+1}, s\in\mathcal{S}. \qquad \text{(1v)}$$

Constraints (1q)–(1v) keep track of ships added to and removed from the fleet. Constraints (1q) set the initial fleet while constraints (1r) ensure that the model keeps track of the delivery of new buildings ordered in the past (i.e., in earlier planning problems). Constraints (1s) ensure the balance of second-hand purchases, sales and demolitions, while constraints (1t)

maintain the balance of new buildings. Notice that second-hand ships and scrappings are added to or removed from the fleet one period after the decision is made, while new buildings are delivered after $\bar{T}_v$ periods. Constraints (1u) make sure that charters out and lay-ups are actually available in the fleet. Finally, constraints (1v) ensure that ships reaching their age limit are scrapped.

$$y^{SH}_{vts} \leq \overline{SH}_{vts} \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{1w}$$

$$y^{SE}_{vts} \leq \overline{SE}_{vts} \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{1x}$$

$$h^{I}_{vts} \leq \overline{CI}_{vts} \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{1y}$$

$$h^{O}_{vts} \leq \overline{CO}_{vts} \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{1z}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} y^{SH}_{vts} \leq \overline{SH}_{ts} \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, s \in \mathcal{S}, \tag{1aa}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} y^{SE}_{vts} \leq \overline{SE}_{ts} \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, s \in \mathcal{S}, \tag{1ab}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} h^{I}_{vts} \leq \overline{CI}_{ts} \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}, \tag{1ac}$$

$$\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} h^{O}_{vts} \leq \overline{CO}_{ts} \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}. \tag{1ad}$$

Constraints (1w) and (1x) impose a limit on the number of second-hand purchases and sales, respectively, for a given type of ship, while constraints (1y) and (1z) impose a limit on the number of charters in and out, respectively, for a given type of ship. Constraints (1aa), (1ab), (1ac) and (1ad) limit the total number of second-hand purchases, sales, charters in and charters out, respectively. Notice that the bounds depend on the specific market in which the company operates.

$$y^{NB}_{vts} \in \mathbb{Z}^+ \qquad t \in \mathcal{T} : t \leq \bar{T} - \bar{T_v}^L, v \in \mathcal{V}^N_{t+T^L}, s \in \mathcal{S}, \tag{1ae}$$

$$y^{SC}_{vts}, y^{SH}_{vts}, y^{SE}_{vts} \in \mathbb{Z}^+ \qquad t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{1af}$$

$$y^{P}_{vts} \in \mathbb{R}^+ \qquad t \in \mathcal{T}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{1ag}$$

$$h^{I}_{vts}, h^{O}_{vts}, l_{vts} \in \mathbb{R}^+ \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, s \in \mathcal{S}, \tag{1ah}$$

$$x_{vrts} \in \mathbb{R}^+ \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, r \in \mathcal{R}_{vt}, s \in \mathcal{S}, \tag{1ai}$$

$$n_{ikts} \in \mathbb{R}^+ \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^C_t, k \in \mathcal{K}, s \in \mathcal{S}, \tag{1aj}$$

$$\delta_{its} \in \{0, 1\} \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^O_t, s \in \mathcal{S}. \tag{1ak}$$

Finally, constraints (1ae)–(1ak) set the domain for the decision variables. Notice that

variables y^P_{vts} are continuous, as their integrality is automatically enforced by constraints (1q)–(1v).

Model (1) is assumed to be nonanticipative, i.e., decisions are only based on current information. This is enforced through so called "nonanticipativity constraints" which are however not shown for the sake of legibility. Alternatively, it is possible to obtain an equivalent *node formulation* of model (1) which implicitly ensures nonanticipative solutions. Such formulation, which associates decisions and realizations of random parameters to the nodes of the underlying scenario tree, is provided in the appendix. Generally, a node formulation yields an optimization problem with significantly fewer decision variables and constraints and is often suitable for solving the corresponding problem by means of a solver. While the node formulation will be used in our computational study, in what follows we continue to refer to the scenario formulation (1) for ease of exposition.

A possible limitation of model (1) is that it tends to become a very large optimization problem as the number of scenarios increases in an attempt to provide a better description of the uncertainty. This is independent of whether the node formulation in the appendix or the scenario formulation (1) is used. As the size of the model increases, specialized algorithms become necessary, see e.g., Pantuso et al. (2015). An additional potential limitation is the high-level description of the sailing operations. In fact, the corresponding fleet deployment problem is in general a complicated optimization problem, see e.g., Powell and Perakis (1997), Fagerholt et al. (2009), Wang and Meng (2012). A simplified set of tactical description is however often required to make strategic decisions. The implications of these simplifications and how they provide a reasonable representation of the sailing operations are discussed in Pantuso et al. (2016). However, the impact of the level of details in short- and mid-term decisions and the quality of long-term decisions is a general open research question beyond the scope of shipping investments.

3.3 Deterministic Cash Flow Control

In this section we introduce a deterministic control mechanism on cash flows. For the sake of legibility, let f^I_{ts} and f^O_{ts} be decision variables representing the cash-inflow and cash-outflow, respectively, in period t and scenario s. For a given time period $t \in \mathcal{T}$ and scenario $s \in \mathcal{S}$, the cash-inflow and cash-outflow are defined as in (2) and (3), respectively.

$$f^I_{ts} = \sum_{i \in \mathcal{N}^O_t} \sum_{k \in \mathcal{K}} R^D_{its} D_{ikts} \delta_{its} + \sum_{i \in \mathcal{N}^C_t} \sum_{k \in \mathcal{K}} R^D_{its} D_{ikts} + \sum_{v \in \mathcal{V}_t} (R^{CO}_{vts} h^O_{vts} + R^{SE}_{vts} y^{SE}_{vts} + R^{LU}_{vts} l_{vts} + R^{SC}_{vts} y^{SC}_{vts}) \tag{2}$$

$$f_{ts}^{O} = \sum_{v \in \mathcal{V}_t^{IN}} C_{vts}^{IN} Y_v^{IP} + \sum_{t-M+1 \le t' \le t} \sum_{v \in \mathcal{V}_t^{IN}} (C_{vt'ts}^{IN} y_{vt's}^{NB} + C_{vt'ts}^{SH} y_{vt's}^{SH}) \\ + \sum_{i \in \mathcal{N}_t^C} \sum_{k \in \mathcal{K}} C_{ikts}^{SP} n_{ikts} + \sum_{v \in \mathcal{V}_t} (C_{vts}^{OP} y_{vts}^{P} + C_{vts}^{CI} h_{vts}^{I} + \sum_{r \in \mathcal{R}_{vt}} C_{vrts}^{TR} x_{vrts}) \tag{3}$$

Thus, the cash-inflow f_{ts}^{I} consists of the revenue from contracted and optional trades, the revenue from scrapping ships, selling and chartering out ships, and the operating expense savings for laying-up ships. The cash-outflow f_{ts}^{O} consists of the instalments paid for the new ships ordered and for the purchases in the second-hand market, the time and space chartering expenses, and the fixed and variable operating expenses.

Furthermore, let $\bar{F}$ be the worst-case cash flow tolerated by the company, and B the budget available for ordering and purchasing ships in the first period (determined by known ongoing expenses generated by the solution to earlier planning problems). cash flows can deterministically (i.e., for all scenarios considered) be controlled by means of the following constraints which can be added to the basic model presented in Section 3.2:

$$B + \sum_{v \in \mathcal{V}_0} \left(R_{v0s}^{SE} y_{v0s}^{SE} + R_{v0s}^{SC} y_{v0s}^{SC} \right) - \sum_{v \in \mathcal{V}_0^{IN}} \left(C_{v00s}^{IN} y_{v0s}^{NB} + C_{v00s}^{SH} y_{v0s}^{SH} \right) \ge \bar{F} \qquad s \in \mathcal{S} \tag{4}$$

$$f_{ts}^{I} - f_{ts}^{O} \ge \bar{F} \qquad t \in \mathcal{T} \setminus \{0\}, s \in \mathcal{S}. \tag{5}$$

Constraints (4) and (5) ensure that cash flows are higher than the specified safety limit in the first and following periods, respectively. Notice that the operating revenues and expenses are not included in period 0 as they are the result of earlier planning problems. Thus, constraints (4) ensure that investments in the first period, given a budget B, do not violate the safety cash flow level $\bar{F}$.

3.4 Conditional Value-at-Risk Cash Flow Control

In this section we extend the model in Section 3.2 to limit the Conditional Value-at-Risk (CVaR), which have been used in a number of applications to control risk, see for example (Xue et al., 2015) and (Xinsheng et al., 2015). The CVaR represents the expected loss in the worst $\alpha\%$ scenarios. We impose constraints on the CVaR in every time period. Two parameters, namely a confidence level and a minimum CVaR value, which are input to our model, reflect the degree of risk aversion held by the shipping company. In our problem, for example, given a confidence level of 95% and a minimum CVaR value of $\$-30M$, the CVaR

constraints would restrict the average cash flow in the worst 5% scenarios in every time period to be above $\$-30M$.

Since we mainly study a two-stage case (even though the model we presented in Section 3.2 can be multistage, depending on the underlying scenario tree), we assume now, to simplify the following explanation, that the MFRP is modeled as a two-stage stochastic program. Let $\mathcal{T}^F \subseteq \mathcal{T}$ be the set of first-stage time periods and $\mathcal{T}^S \subseteq \mathcal{T}$ be the set of time periods in the second-stage. Let $\alpha \in [0,1]$ be the confidence level, and ζ and η_{ts} artificial variables necessary in the CVaR constraints. It can be shown that variable ζ, at the optimal solution, represents the Value-at-Risk (VaR), see Rockafellar and Uryasev (1997). Variables η_{ts} represent the negative cash flows in excess of VaR in period t and scenario s. Finally, let $\bar{F}_\alpha$ be the minimum allowed expected cash flow under confidence level α.

We adapt the constraints (4) and (5) to control cash flows in all first-stage periods $\mathcal{T}^F$. For the periods affected by uncertainty, i.e. the time periods in $\mathcal{T}^S$, we limit the CVaR by applying the following constraints.

$$\zeta + \frac{1}{1-\alpha} \sum_{s \in \mathcal{S}} P_s \eta_{ts} \geq \bar{F}_\alpha \qquad t \in \mathcal{T}^S, \tag{6}$$

$$\eta_{ts} \leq f^I_{ts} - f^O_{ts} - \zeta \qquad t \in \mathcal{T}^S, s \in \mathcal{S}, \tag{7}$$

$$\eta_{ts} \in \mathbb{R}^- \cup 0 \qquad t \in \mathcal{T}^S, s \in \mathcal{S}. \tag{8}$$

Notice that artificial variables η_{ts} take non-positive values and that when the cash flow is short of VaR, the artificial variable η_{ts} becomes negative and is included in constraints (6) which compute and bound the value of CVaR. Notice also that the deterministic cash flow control constraints introduced in Section 3.3 are a special case of constraints (6)–(8) with a sufficiently high confidence level α. For example, if the number of scenarios, $|\mathcal{S}|$, is equal to 100 and $\alpha = 0.99$, the expected cash flow of the $(1 - 0.99) * 100$ worst scenarios corresponds to the cash flow of *the* worst scenario. In this case, bounding CVaR is equivalent to imposing a deterministic bound on cash flows.

4 Computational Study

The scope of this computational study is to test the alternative cash flow control models introduced in Section 3 on instances based on data from a real shipping company. Particularly, we focus on understanding the trade-off between expected profits and protection against adverse market scenarios.

The models introduced in Section 3 (particularly their equivalent node formulations pre-

sented in the appendix) were implemented using IBM ILOG CPLEX 12.6.1 C++ callable libraries. Tests were performed on a computer equipped with an Intel ® Core ™i7-4500U CPU @ 1.8 GHz (2.4 Ghz) and 8 GB RAM.

4.1 Instances

We use three instances, named *Small*, *Medium* and *Large*, adapted from Mørch et al. (2017) and based on data from a major shipping company which operates in the RoRo shipping market. The three instances represent three shipping companies of different sizes. The underlying characteristics of the ships and trades are identical to Mørch et al. (2017). However, we adjusted the initial fleet and considered a different subset of the available trades with the scope of observing the trade-off risk aversion-profits in companies of different size. The initial fleet and trades for the three instances are reported in Table 2 and Table 3, respectively. The RoRo market is characterized by highly specialized ships due to the specific technology required to load, unload, and host rolling equipment. Consequently, the second-hand market and the market for charters are rather small. Therefore, second-hand purchases and sales, and well as charters in and out are excluded from the model when running the tests. As a consequence, fewer options are available to adapt the fleet to ongoing market conditions, and the importance of planning against uncertainty is emphasized (see, e.g., the discussion in Pantuso et al. (2016)). In the basic settings, the models are implemented as two-stage models. In Section 4.4 we show that this is an acceptable simplification of the real-life problem. The three instances have a planning horizon of five years.

For each instance, 18 ship types are available. However, the size and composition of the initial fleet, as well as the trades serviced, vary between the instances. Tables 2 and 3 report the ship classes and trades in the three instances, respectively. A ship class represents the technical specifications of the ship. However, different ship types are obtained from each ship class, depending on the age of the available ships. The company transports three types of products, namely cars, High & Heavy vehicles (HH – mainly agriculture and industrial vehicles) and Break-Bulk cargo (BB – items with high volume or weight such as train coaches or big engines). The measurement unit for the three type of products is RT43, a standard unit in the RoRo shipping business. The ship types operated can carry the three types of products, but in different proportions. The capacities of the available ship types are reported in Table 2 while the demands are reported in Table 3. Note that the total capacity of a ship from a given class corresponds to the highest of the individual capacities as the compartments of the ships are not necessarily dedicated to a specific type of product. The large instance represents a shipping company with 51 ships (see Table 2) in the initial fleet servicing 11 to

Table 2: Ship types and respective capacities in the instances

	Initial fleet			Capacity			
Ship Class	Small	Medium	Large	Cars	HH	BB	Total
PCTC1	2	3	5	4975	2200	300	4975
PCTC2	5	10	15	6800	2500	300	6800
PCTC3	2	9	12	5450	2200	900	5450
PCTC4	1	5	6	6150	1800	200	6150
LCTC1	5	8	9	6000	2000	1500	6000
RORO1	1	2	4	4853	3100	1500	4853
RORO2	0	0	0	5660	4000	2200	5660
Total	16	37	51				

14 trades (see Table 3) with a total demand for the first year of approximately 2.9M RT43. In the medium instance there are 37 ships and seven to nine trades. The resulting total demand is thus approximately 65% of the demand in the large instance. The small instance has 16 ships and three to five trades with a total demand of approximately 30% of the demand in the large instance. However, in all instances the demand can be increased by approximately 10% by means of optional trades.

Table 3: Trades in the instances. Demands are for the first year. Labels "C" and "O" indicate contracted and optional trades, respectively, while a dash ("-") indicates that the trade is not included in the instance.

Trades	BB	Car	HH	Small	Medium	Large
AFEU	0	19 200	0	O	C	C
ASCE	2 166	119 397	44 120	-	-	C
ASEU	5 761	435 213	77 046	C	C	C
ASNE	36 845	35 331	66 606	C	O	C
ASNW	1 939	60 158	7 400	O	O	O
EUNAOC	26 198	297 688	123 779	-	C	C
EUNE	20 075	469 379	67 853	-	-	C
EUNW	7 425	89 405	21 427	-	C	C
EUOC	16 776	266 855	55 474	-	C	C
NAAS	3 251	24 818	10 434	-	-	O
NAEU	14 115	140 508	53 928	-	-	O
NAME	4 476	103 048	14 200	-	C	C
NASA	3 404	41 036	24 419	-	-	C
SANA	2 689	97 999	29 239	C	C	C

With respect to the other parameters, the lead time $\bar{T}_v^L$ is set to two years. The instalments paid in each period are determined by the new building price, the repayment time and the

interest rates offered by banks or the expected return on investments from investors. We set a five years repayment time. This is consistent with Stopford (2009) who states that the repayment time is normally between two and eight years. Moreover, Stopford (2009) states that the interest rates on loans for financing investments in ships are generally quoted at a margin over *London Interbank Offered Rate* (LIBOR). The spread of this margin is typically in the range 0.6% to 2.0%. Therefore, 1.25% is chosen as a margin on top of the LIBOR. Using the June 2017 one-year LIBOR rate of 1.75%, we obtain an interest rate of 3.0%.

The progression of the value of the ships in the instances is estimated using a linear depreciation based on the new building cost and an expected lifetime of 30 years. This is consistent with the findings in Stopford (2009) for the Panamax bulk carriers sold in the first nine months of 2002. The sunset value is set to 70% of the ship value in the last period, a value that, after preliminary testing, was found to be sufficient for preventing over-investments, while providing the desired modeling feature sunset values are intended to have, i.e. to maintain a realistic fleet at the end of the planning horizon.

Space charter prices are set to 2000 USD per RT43, which was shown to give a reasonable and realistic use of space charters, while at the same time reasonably close to the real value of using such an option. This can also be considered a penalty cost for unsatisfied demand, and thus the parameter is considered deterministic. As suggested by Stopford (2009), all input values are properly discounted using a discount factor of 12% to ensure that decisions made early in the planning horizon become more important than later decisions. Finally, the budget for ordering ships in year 0 is set to be 5% of the contracted revenue in period 1, assuming similar revenues in periods 0 and 1.

Uncertainty is modeled by associating a random variable to each stochastic parameter in the problem. Particularly, we include one random variable representing the demand of each of the three products on each trade, one random variable for ship prices, one random variable for the fuel price (influencing sailing costs), and one random variable for steel price (influencing scrapping revenues). For each of these elements we assume a triangular distribution such that one can have a one-year change in the range -50% to +50%. The correlations between the random variables are shown in Table 4. We assume that the first two years belong to the first stage, thus $\mathcal{T}^F = \{0, 1, 2\}$, while the remaining periods belong to the second stage, thus $\mathcal{T}^S = \{3, 4, 5\}$. Scenarios for the random variables are generated using the method provided by Kaut and Lium (2014), which uses distribution functions and correlations. We achieve acceptable in-sample stability (see Kaut and Wallace (2007)) with 100 scenarios.

Table 4: Correlation matrix

		Trade 1			Trade 2	···	Trade N			
		Car	HH	BB	Car	···	BB	New building price	Fuel	Steel
	Car	1	0.7	0.7	0.7	···	0.7	0.7	0.2	0.2
Trade 1	HH		1	0.7	0.7	···	0.7	0.7	0.2	0.2
	BB			1	0.7	···	0.7	0.7	0.2	0.2
Trade 2	Car				1	···	0.7	0.7	0.2	0.2
⋮	⋮					⋱	⋮	⋮	⋮	⋮
Trade N	BB						1	0.7	0.2	0.2
	New building price							1	0.2	0.2
	Fuel								1	0.2
	Steel									1

As previously mentioned, space charter costs can also be seen as penalty for unfulfilled demand, which in some cases is difficult to quantify. Similarly, the correlation between the random variables might be difficult to estimate, e.g., when historical data is scarce. Therefore, we test four different versions of the problem, namely for the combinations with normal and 50% reduced space charter cost, and with either all random variables correlated as shown in Table 4 or with no correlation between them (i.e., with correlation matrix corresponding to an identity matrix of suitable dimensions).

4.2 Effects of Deterministic Cash Flow Control

In this section we test the effect of using the deterministic cash flow control introduced in Section 3.3. We start by showing the results for the base case where we assume normal space charter costs and correlated random variables. Particularly, we solve the basic model presented in Section 3.2 with the addition of constraints (4)–(5) for different values of cash flow limit $\bar{F}$. We start by solving the large instance without cash flow control (corresponding to a risk neutral decision maker) and observe the worst-case cash flow. Then, we set $\bar{F}$ at this value and solve the problem with increased values of $\bar{F}$, stopping when an infeasible problem is obtained. Table 5 reports a summary of the first- and second-stage solutions obtained for different levels of $\bar{F}$, where the first row represents the solution without cash flow constraints.

It can be noticed that the expected profit decreases with increasing $\bar{F}$ (except for some noise due to the 1% optimality tolerance – see for example the increase from the third row to the fourth row). However, in general, we observe that a significant increase of the worst-case cash flow can be obtained at the price of only a negligible reduction of the expected profit. In fact, the worst-case cash flow can be increased from $\$ - 66.3M$ to $\$ - 39.8M$ almost without reducing the expected profit. This is due to the fact that the problem has a flat objective function with many near-optimal solutions. However, when stricter control on cash flows is imposed, a significant reduction of expected profits is registered. It can be noticed that

stricter cash flow limits are dealt with by reducing the number of new buildings, using the available ships for longer times (see the reduced scrappings) and using more space-charters. This corresponds to saying that the installments associated with new buildings are a major cause of negative cash flows. Finally, optional trades are used to increase the total demand when the demand is low, and they are not serviced at all in the high demand scenarios.

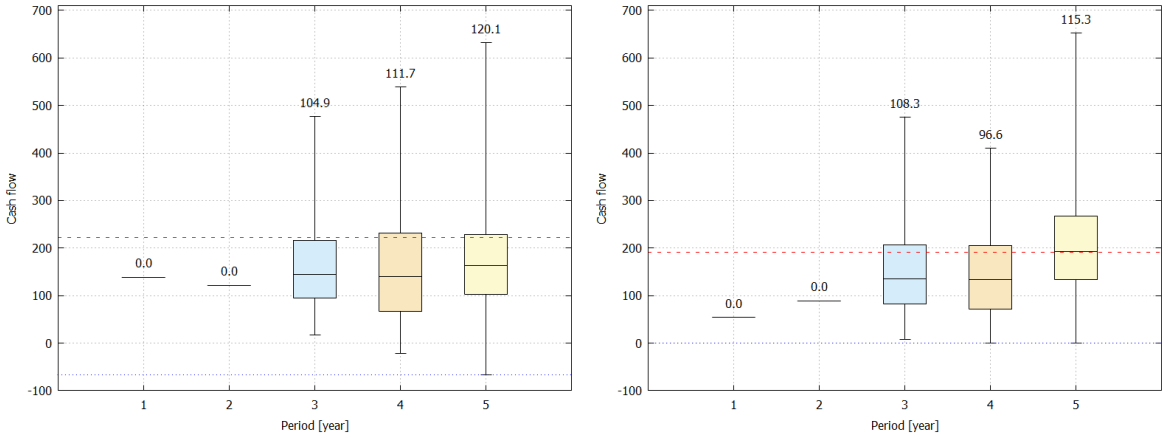
Table 5: Solutions to instance Large for increasing cash flow limits $\bar{F}$ with correlated random variables and normal space charter prices. Columns named 1^{st} and 2^{nd} report summaries of first- and second-stage solutions, respectively. The numbers for the second-stage decisions are average values over the 100 scenarios.

$\bar{F}$	Expected	New buildings		Scrappings		Lay-up		Space		Optional Trades	
[$M]	Profit [$M]	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}
-66.3	1 109.4	29	1.99	9	19.09	0.37	12.90	0	9 924	0	1.55
-59.7	1 109.0	29	2.00	9	19.19	0.37	12.72	0	13 989	0	1.46
-53.1	1 108.5	29	1.98	9	19.16	0.37	12.84	0	10 913	0	1.55
-46.4	1 110.1	29	2.08	9	19.28	0.37	12.61	0	12 416	0	1.47
-39.8	1 109.3	29	2.02	9	19.16	0.37	12.72	0	12 912	0	1.48
-33.2	1 104.9	26	2.39	8	17.85	0.41	12.56	0	26 709	0	1.42
-26.5	1 094.2	26	2.41	9	16.77	2.37	12.63	0	35 091	0	1.46
-19.9	1 078.6	26	2.39	9	16.86	4.36	12.69	0	34 769	0	1.46
-13.3	1 065.4	26	2.37	9	16.71	6.34	12.84	0	38 723	0	1.52
-6.6	1 038.2	24	2.70	6	18.16	10.39	12.64	0	28 493	0	1.44
0.0	955.7	23	2.67	5	18.34	12.19	15.05	42 367	36 051	0	1.54

Figure 3 shows the cash flow development for each period and scenario as a box plot comparing the solution of the model without cash flow constraints (Figure 3a) and the solution of the model with the tightest cash flow constraints (Figure 3b). These solutions correspond to $\bar{F} = -66.3$ and $\bar{F} = 0$ in Table 5. The red dashed line represents the annualised expected profit. The lower end of the box represents the first quartile, the upper end of the box represents the third quartile, and the line inside the box represents the median. The ends of the whiskers represent the minimum and maximum net cash flows. This means that 50% of the scenarios are located inside the box, while 25% is located on each side of the box between the ends of the box and the ends of the whiskers.

In Figure 3a and Figure 3b it can be noticed how the worst-case cash flow, i.e. the bottom whisker in year 5 without cash flow control, is improved when controlling cash flows, at the cost of expected profit loss and a reduction of the best-case cash flow. Furthermore, we can observe a reduction in the cash flow standard deviation in periods 4 and 5 of Figure 3b.

To interpret and visualize the four versions of the large instance, i.e., with and without reduced space charter costs and with and without correlated random variables, the four effi-



(a) Without cash flow constraints **(b)** With cash flow constraints ($\bar{F} = 0$)

Figure 3: The cash flow development for the large instance with correlated random variables and normal space charter price. The red dashed line is the annualized expected profit, and the numbers above each whisker is the standard deviation for the given period.

cient frontiers are plotted in Figure 4. The efficient frontiers show the profit loss (compared to the risk neutral case) generated by different values of the cash flow limit $\bar{F}$. Note that the decreasing relative expected profit loss in parts of the curves are the result of the 1% optimality gap, and not representing the real situation. Thus, in reality the curves are always non-decreasing if the instances are solved to optimality.

From Figure 4 it is clear that all versions of the large instance have the same characteristic. There exists a portion of the curves where the worst-case cash flow can be increased with only small losses in expected profit, and a portion where the expected profit loss is rapidly increasing with the worst-case cash flow. Furthermore, when tightening up the cash flow limits there is a significant difference between the uncorrelated and correlated versions with normal space charter price. This illustrates that in the real-world (where there exists some positive correlation between the random variables involved) the benefit of using the cash flow control model is greater than in an uncorrelated world. For example, the relative worst-case cash flow of -10% has an expected profit loss of approximately 6% and 2% for the uncorrelated and correlated versions, respectively. In addition, the worst-case cash flow can be improved by approximately 15% in terms of the annualized expected profit without any significant loss in expected profit.

We can also see similar effects for the small and medium instances as for the large one. Figures 5a and 5b show the efficient frontiers for the four different versions of the small and medium instances, respectively. Also here, it is possible to significantly improve the worst-case cash flow without much loss in expected profit in most versions. However, we see that for

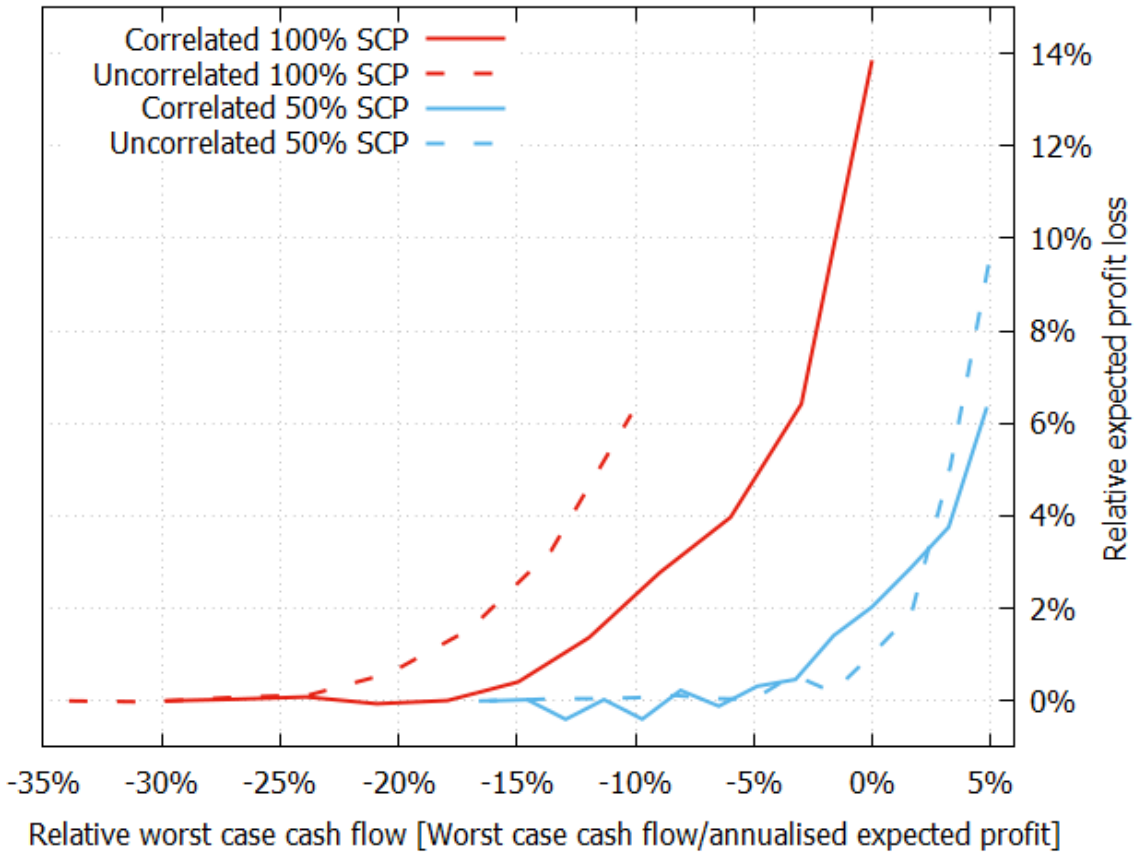
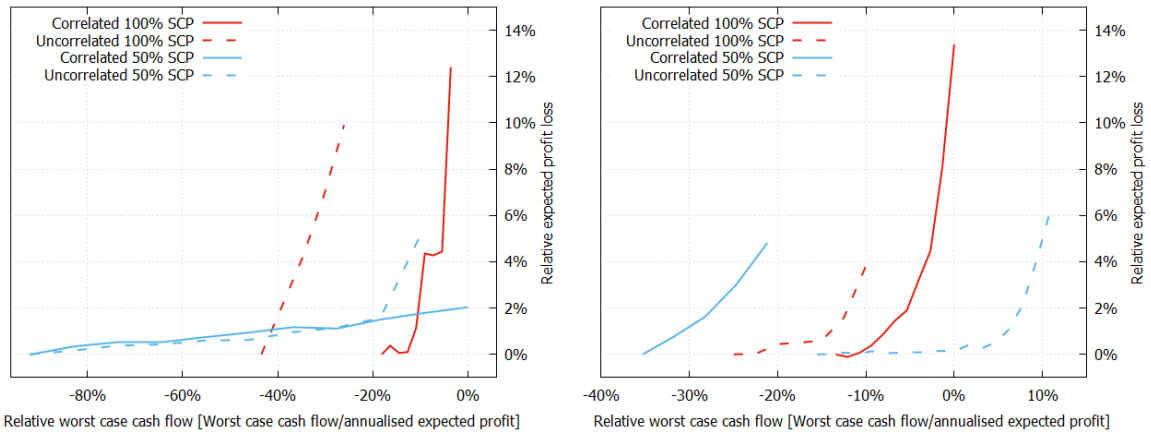


Figure 4: The efficient froniters for the different versions of the large instance. Let E^* be the annualized expected profit without cash flow control, and $E^{\bar{F}}$ the annualized expected profit using cash flow threshold $\bar{F}$. The vertical axis represents the ratio $100 * (E^* - E^{\bar{F}})/E^*$. Let $WC^{\bar{F}}$ be the worst-case cash flow obtained for a given threshold $\bar{F}$. The horizontal axis represents the ratio $100 * WC^{\bar{F}}/E^*$. Finally, SCP is an abbreviation for space charter price.

some versions, such as both versions of the small instance with normal space charter price and both correlated versions for the medium instance, there is not that much room for improving the worst-case cash flow without large losses in expected profit.

The reason we see different shapes of the efficient frontiers for the small and medium instances (Figure 5) compared to the large (Figure 4) is that, the smaller the instance gets, the higher is the relative impact of a decision. Note how the efficient frontiers for the large instance can be represented by piecewise linear functions with an increasing gradient while the efficient frontier for some of the small instances only consists of one linear function, corresponding to less flexibility. These linear sections of the efficient frontier for the small instance also appear to have a longer range than for the large instance, but one must recall that the cost of buying a ship compared to the expected profit is relatively higher in the small



(a) Small instance **(b)** Medium instance

Figure 5: The efficient froniters for the different versions of the small and medium instances. Let E^* be the annualized expected profit without cash flow control, and $E^{\bar{F}}$ the annualized expected profit using cash flow threshold $\bar{F}$. The vertical axis represents the ratio $100 * (E^* - E^{\bar{F}})/E^*$. Let $WC^{\bar{F}}$ be the worst-case cash flow obtained for a given threshold $\bar{F}$. The horizontal axis represents the ratio $100 * WC^{\bar{F}}/E^*$. Finally, SCP is an abbreviation for space charter price.

instance compared to the large instance.

4.3 Effects of CVaR Cash Flow Control

We tested the CVaR cash flow control model presented in Section 3.4 with confidence levels of $\alpha = 0.99$, 0.95 and 0.90 on the large instance with correlations as shown in Table 4 and normal space charter price. Note that since the instance is solved with 100 scenarios, the CVaR with $\alpha = 0.99$ is equivalent to the deterministic cash flow control model from Section 3.3.

Table 6 presents the solutions for the CVaR model with confidence level of 0.95. When limiting the expected cash flow of the 5% worst scenarios, the expected profit loss is much lower than for $\alpha = 0.99$ (i.e., the deterministic cash flow case, see Table 5). This can also be seen by comparing the efficient frontiers in Figure 6. An immediate observation is that, as intuition suggests, a higher tolerance of risk leads to higher profits. As an example, when the relative expected cash flow limit is 0%, a risk tolerance corresponding to $\alpha = 0.95$ leads to a profit loss of approximately 2% compared to a risk neutral decision maker. However, a lower risk tolerance (corresponding to $\alpha = 0.99$) yields a profit loss of approximately 14% for the same relative expected cash flow limit. Therefore, a decision maker willing to limit the negative expected cash flows in the worst 5% scenarios, rather than in the worst 1% (corresponding to a more strict policy) is rewarded with a significantly higher expected profit,

Table 6: Solutions for the large instance with correlated random variables and normal space charter prices for the CVaR model with $\alpha = 0.95$.

$\bar{F}_\alpha$	Expected	New builds		Scrappings		Lay-up		Space		Optional	
[$M]	Profit [$M]	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}
-66.3	1109.4	29	1.99	9	19.09	0.37	12.9	0	9 924	0	1.55
-59.7	1109.1	29	2.00	9	19.16	0.37	12.85	0	10 162	0	1.54
-53.1	1109.9	30	1.99	9	19.58	0.37	13.70	0	454	0	1.46
-46.4	1110.4	29	1.99	9	19.10	0.37	12.80	0	12 184	0	1.46
-39.8	1109.1	29	2.12	9	19.30	0.37	12.66	0	12 998	0	1.46
-33.2	1110.0	29	2.28	9	19.35	0.37	12.57	0	11 082	0	1.47
-26.5	1109.4	28	2.19	8	19.34	0.41	12.62	0	12 642	0	1.45
-19.9	1108.6	29	1.99	9	19.11	0.37	12.91	0	10 713	0	1.55
-13.3	1109.1	29	2.01	8	20.08	0.41	12.76	0	5 811	0	1.46
-6.6	1108.2	27	2.26	9	17.58	0.37	12.66	0	26 938	0	1.45
0	1093.7	26	2.39	8	17.86	2.37	12.62	0	26 128	0	1.45
6.6	1078.2	27	2.43	9	17.91	4.41	12.71	0	26 004	0	1.51
13.3	1033.0	23	3.05	4	19.89	8.27	12.69	5 992	32 448	0	1.40

corresponding to only a 2% loss compared to that of a risk neutral decision maker. It can be noticed that, in the left-hand-side portion of the efficient frontier, the increase in the cash flow does not result in expected profit losses. This means that, independently of the degree of risk aversion of the decision maker, there is the possibility of significantly limiting the risk of negative cash flows while ensuring approximately the same expected profit as that of a risk neutral decision maker. However, in the right-hand-side portion of the efficient frontier, the difference between different degrees of risk aversion leads to significantly different expected profits.

4.4 From a Two-stage to a Three-stage Model

The instances solved in Sections 4.2 and 4.3 have been solved using a two-stage model even though a multi-stage representation is clearly closer to the reality. In this section we compare the results between a two-stage and a three-stage representation of the problem to examine whether the former is a reasonable simplification. For computational reasons we run tests only on the small instance. Furthermore, we use the uncorrelated settings since the correlated setting requires a higher number of scenarios for the scenario generation algorithm to work correctly, resulting in an excessive computation time for the three-stage model. Finally, we use a 50% reduction in space charter prices as it provides a wider range where the worst-case cash flow can be improved, as seen in Figure 5a. In the three-stage model, the decision stages are period 1, 3 and 5. At every stage we generate 20 conditional realizations, resulting

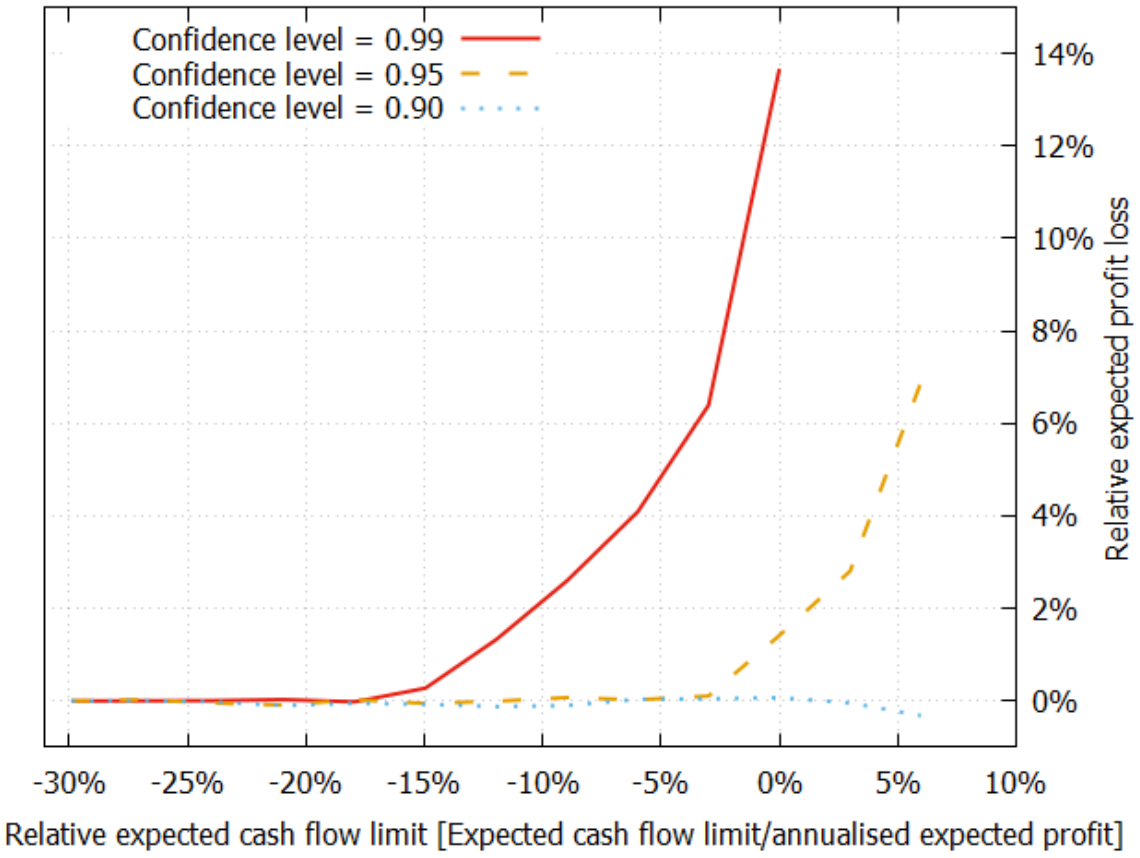


Figure 6: The efficient frontiers for the CVaR model solved for the large instance with correlated random variables and normal space charter price. Let E^* be the annualized expected profit without cash flow control, and $E^{\bar{F}_\alpha}$ the annualized expected profit using cash flow threshold $\bar{F}_\alpha$. The vertical axis represents the ratio $100 * (E^* - E^{\bar{F}_\alpha})/E^*$. The relative expected cash flow limit is calculated as the minimum expected cash flow allowed, $\bar{F}_\alpha$, divided by the annualized expected profit.

in a total of $20 * 20 = 400$ scenarios. The efficient frontiers for the three- and two-stage solutions are shown in Figure 7. They both have similar characteristics with a section where the worst-case cash flow is increased at a small cost in expected profit loss, and a section where the cost is rapidly increasing with worst-case cash flow improvement. This indicates that the characteristics found in the efficient frontiers are similar between the two-stage and three-stage model versions. Therefore, the two-stage simplification seems to give a good trade-off between computational time and solution quality, at least for the small instance with uncorrelated random variables and reduced space charter price.

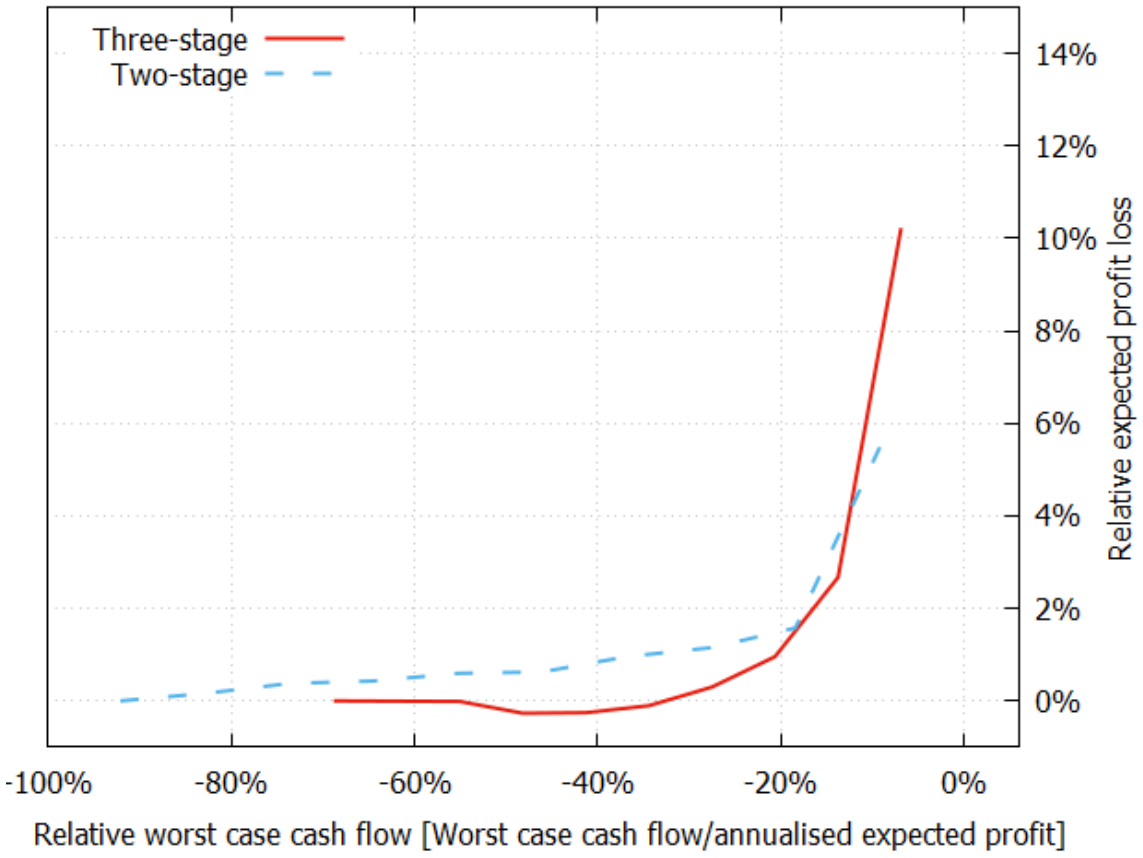


Figure 7: The efficient frontiers for the three and two stage solutions for the small instance with uncorrelated correlation random variables and reduced space charter price. Let E^* be the annualized expected profit without cash flow control, and $E^{\bar{F}}$ the annualized expected profit using cash flow threshold $\bar{F}$. The vertical axis represents the ratio $100 * (E^* - E^{\bar{F}})/E^*$. Let $WC^{\bar{F}}$ be the worst-case cash flow obtained for a given threshold $\bar{F}$. The horizontal axis represents the ratio $100 * WC^{\bar{F}}/E^*$.

4.5 Discussion and Managerial Insights

To determine which confidence level and solution a company should choose depends on their current situation and risk preferences. A company's utility of a solution might change whether they face the risk of cash flow insolvency or balance-sheet insolvency. If the company is low on cash reserves and thus is facing an immediate risk of cash flow insolvency, the manager will probably choose a solution from the steep part of the efficient frontier, where the limit on the worst-case cash flow is stricter and thus the protection against this type of risk is stronger.

On the other hand, if the company has a big cash reserve for the coming planning period, but for some reason the value of their assets (for instance ships) dropped to a level where their liabilities are greater than the asset values, the company would be facing the risk of balance-sheet insolvency. In this situation the manager would probably choose a solution further to

the left on the efficient frontier in order to maximize the expected profit, thus increasing the value of the company and reducing the risk of balance-sheet insolvency. However, this will come at the cost of increased risk of low-cash flows in a poor market.

A risk neutral decision maker will probably maximize expected profit regardless of the risk, thus not using a binding confidence level at all (corresponding to a solution from the efficient frontiers with lower confidence levels, e.g., the bottom curve in Figure 6). A more risk averse decision maker will probably, for a given relative expected cash flow limit, prefer a solution with a higher confidence level. On the other hand, for a given confidence level, the risk averse decision maker will probably choose a solution more to the right of the given efficient frontier to increase the relative expected cash flow limit, though perhaps at a cost of a loss in expected profit. However, the solutions between -30% and -15% in relative expected cash flow limit in Figure 6 are close to indifferent with respect to expected profit. Hence, one can hedge against bad cash flow periods with very little expected profit loss by choosing a solution from an efficient frontier with a higher confidence level or by increasing the relative expected cash flow limit.

The results presented in the previous sections show that in most cases it is possible to reduce the risk (i.e. the relative worst case cash flow) significantly with very little loss in expected profit. Furthermore, the results show that stricter cash flow limits or higher risk aversion are dealt with by reducing the number of new buildings, using the available ships for longer times (i.e. reducing the number of scrappings) and using more space-charters. This corresponds to saying that the installments associated with new buildings are a major cause of negative cash flows. Finally, optional trades are used to increase the total demand when the demand is low, and they are not serviced at all in the high demand scenarios.

The CVaR model provides the decision maker with a tool to choose a risk level matching their situation and risk preference. The deterministic cash flow control is more conservative, but has the advantage that it might be easier to use and interpret its results for a manager of a shipping company. The computational study in this paper demonstrates that the CVaR model can serve as a valuable decision making tool for a risk averse decision maker with the following highlighted benefits:

- The decision maker can explicitly define their risk preferences by adjusting the confidence level and the cash flow threshold.
- There exists a great potential of finding solutions that will allow the company to hedge against periods with bad cash flows without compromising expected profit.
- Constraining the CVaR does not significantly increase the complexity of the model with

respect to a risk neutral setting.

5 Concluding Remarks

We introduced two new models for solving the Maritime Fleet Renewal Problem (MFRP) focusing on controlling the risk of insolvency. The first is a deterministic cash flow control model, while the second model uses Conditional Value-at-Risk (CVaR) constraints to control the risk. In both models, the payments of ships are modeled as instalments rather than lump sums to capture the cash flows more precisely. The deterministic cash flow control model is shown to be a special case of the CVaR model having such a high confidence level that just one worst-case scenario is controlled.

The computational study demonstrated how a shipping company can use the two proposed models to provide decision support in assessing the trade-offs between risk and expected profits. It was shown that solutions of the deterministic cash flow control model for increasing cash flow limits improve the cash flow in the worst-case scenario. However, this comes at the cost of reduced expected profit. Furthermore, by solving the CVaR model for a set of confidence levels the company can adjust their risk level according to their risk preference.

In the case study in this paper, we looked at the roll-on roll-off shipping segment, where the possibility of using secondhand ships and charters is limited and was therefore not considered in the tests (although the proposed model includes it). In other shipping segments, these possibilities are more prominent and should be included. It is expected that having such possibilities would reduce the need of controlling the cash flows as the charter and second-hand markets provide additional recourse actions which can be used to reduce the consequences of unfavorable first-stage decisions. However, further research and tests are required to verify this expected behaviour.

Another direction for future research could be to include cash flow reserves. In this paper, we have assumed that there exists an internal cash flow calculated by the company. When this is not the case, cash flow reserves could be introduced to endogenously determine the appropriate cash flow limit. That is, profits may be used as a cash reserve to prepare for future unfavorable markets.

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Appendix - Node formulation

Sets	
$\mathcal{T}$	Set of periods, indexed by t
$\mathcal{L}$	Set of nodes, indexed by n
$\mathcal{L}_t$	Set of nodes in a time period t, indexed by n. $a(n,t')$ is the ancestor node of node n in the scenario tree in period t', with $a(n,t-1)$ written as $a(n)$.
$\mathcal{K}$	Set of products, indexed by k
$\mathcal{V}_t$	Set of ship types existing in the market in period t, indexed by v
$\mathcal{V}_t^N$	Set of new ship types existing in the market in period t
$\mathcal{V}_t^{IN}$	The set of ship types the company pays instalments for in period t
$\mathcal{N}_t$	Set of trades operated in period t, indexed by i
$\mathcal{N}_t^C$	Set of contractual trades the shipping company is committed to serve in period t, indexed by i
$\mathcal{N}_t^O$	Set of optional trades the shipping company can choose to undertake or not in period t, indexed by i
$\mathcal{R}_t$	Set of loops in period t, indexed by r
$\mathcal{R}_{vt}$	Set of loops that can be sailed by a ship of type v in period t , indexed by r
$\mathcal{R}_{ivt}$	The set of loops servicing trade i that can be sailed in period t by a ship of type v, indexed by r
Parameters	
P_n	The probability for node n to occur
R_{in}^D	The revenue of transporting one unit of goods on trade i, at node n
R_{vn}^{SE}	The selling price for a ship of type v, at node n
R_{vn}^{SC}	The scrapping value of a ship of type v, at node n
R_{vn}^{LU}	The lay-up savings for one period, for a ship of type v, at node n
R_{vn}^{SV}	The sunset value of a ship of type v, at node n
R_{vn}^{CO}	The charter out revenue for one period , for a ship of type v, at node n
C_{vn}^{CI}	The charter in cost for a ship of type v, at node n
R_{vn}^{CO}	The charter out revenue for a ship of v, at node n
C_{vn}^{OP}	The fixed operating cost for a ship of type v, at node n
C_{vrn}^{TR}	The cost of performing a loop r, for a ship of type v, at node n
C_{ikn}^{SP}	The space charter cost for one unit of product k on trade i , at node n
$\overline{CI}_{vn}$	The limit on number of ships of type v available for chartering in at node n
$\overline{CO}_{vn}$	The limit on number of ships of type v available for chartering out at node n

$\overline{SH}_{vn}$	The limit on number of ships of type v available for purchase in the second hand market at node n
$\overline{SE}_{vn}$	The limit on number of ships of type v that can be sold in the second hand market at node n
$\overline{CI}_n$	The limit of the total number of ships that can be chartered in at node n
$\overline{CO}_n$	The limit of the total number of ships that can be chartered out at node n
$\overline{SH}_n$	The limit of the total number of ships that can be bought in the second hand market at node n
$\overline{SE}_n$	The limit of the total number of ships that can be sold in the second hand market at node n
$\bar{T}$	The last time period in the planning horizon
$\bar{T}_v^L$	The lead time for building a ship of type v
Q_{vk}	The total capacity of product k on ship of type v
Q_v	The total capacity on ship of type v
Z_{vr}	The time a ship of type v needs to perform a loop r
Z_v	The total available time in one period for a ship of type v
D_{ikn}	The demand on trade i of product k in node n
F_{in}	The frequency requirement on trade i in node n
Y_{vn}^{NB}	The number of ships of type v ordered in the previous planning period, delivered at node n in the beginning of the time period
Y_v^{IP}	The initial fleet of ships of type v
$C_{va(n,t')n}^{IN}$	The instalment paid at node n on a ship of type v ordered at node $a(n,t')$
C_{vn}^{IN}	The instalment paid for a ship of type v at node n for the ships in the initial fleet, i.e before the planning horizon begins
$C_{va(n,t')n}^{SH}$	The instalments paid at node n on a ship of type v bought in the second hand market at node $a(n,t')$
M	Number of instalments
$\bar{F}$	The cash flow limit, i.e. no period and scenario are allowed to have a worse cash flow than this limit
B	The budget available for ordering or purchasing ships in period 0
Variables	
y_{vn}^{NB}	The number of new buildings ordered of ship of type v, at node n
y_{vn}^{SH}	The number of ships of type v bought in the second hand market, at node n
y_{vn}^{SE}	The number of ships of type v sold in the second hand market at node n
y_{vn}^{SC}	The number of ships of type v scrapped at node n

y^P_{vn}	The number of ships of type v in the pool, at node n
h^I_{vn}	The number of ships of type v chartered in, at period n
h^O_{vn}	The number of ships of type v chartered out at node n
l_{vn}	The number of ships of type v on lay-up, at node n
x_{vrn}	The number of loops r performed by a ship of type v, at node n
n_{ikn}	The amount of product k delivered at node n, by space charters on trade i
δ_{in}	Set to 1 if the company services optional trade i at node n, 0 otherwise.
f^I_n	The cash inflow at node n
f^O_n	The cash outflow at node n

Cash flow expressions

$$f^I_n = \sum_{i\in\mathcal{N}^O_t}\sum_{k\in\mathcal{K}} R^D_{in}D_{ikn}\delta_{in} + \sum_{i\in\mathcal{N}^C_t}\sum_{k\in\mathcal{K}} R^D_{in}D_{ikn}$$
$$+ \sum_{v\in\mathcal{V}_t}(R^{CO}_{vn}h^O_{vn} + R^{SE}_{vn}y^{SE}_{vn} + R^{LU}_{vn}l_{vn} + R^{SC}_{vn}y^{SC}_{vn}), \qquad t\in\mathcal{T}\setminus\{0\}, n\in\mathcal{L}_t$$

$$f^O_n = \sum_{v\in\mathcal{V}^{IN}_t} C^{IN}_{vn}Y^{IP}_v + \sum_{t-M\le t'\le t}\sum_{v\in\mathcal{V}^{IN}_t}(C^{IN}_{va(n,t')n}y^{NB}_{va(n,t')n} + C^{SH}_{va(n,t')n}y^{SH}_{va(n,t')n})$$
$$+ \sum_{i\in\mathcal{N}^C_t}\sum_{k\in\mathcal{K}} C^{SP}_{ikn}n_{ikn} + \sum_{v\in\mathcal{V}_t}(C^{OP}_{vn}y^P_{vn} + C^{CI}_{vn}h^I_{vn} + \sum_{r\in\mathcal{R}_{vt}} C^{TR}_{vrn}x_{vrn}), \qquad t\in\mathcal{T}\setminus\{0\}, n\in\mathcal{L}_t$$

Objective function

$$\begin{aligned}
max\ z = \sum_{t\in\mathcal{T}\setminus\{0\}}\sum_{n\in\mathcal{L}_t} P_n\Big(&\sum_{i\in\mathcal{N}^O_t}\sum_{k\in\mathcal{K}} R^D_{in}D_{ikn}\delta_{in}\\
&+\sum_{i\in\mathcal{N}^C_t}\sum_{k\in\mathcal{K}}(R^D_{in}D_{ikn} - C^{SP}_{ikn}n_{ikn})\\
&-\sum_{v\in\mathcal{V}_t}(C^{OP}_{vn}y^P_{vn} + C^{CI}_{vn}h^I_{vn} - R^{CO}_{vn}h^O_{vn}\\
&+\sum_{r\in\mathcal{R}_{vt}} C^{TR}_{vrn}x_{vrn} - R^{LU}_{vn}l_{vn})\Big)\\
&-\sum_{t\le M-1}\sum_{n\in\mathcal{L}_t}\sum_{v\in\mathcal{V}^{IN}_t} P_nC^{IN}_{vn}Y^{IP}_v\\
&-\sum_{t\in\mathcal{T}}\sum_{t-M\le t'\le t}\sum_{n\in\mathcal{L}_t}\sum_{v\in\mathcal{V}^{IN}_t} P_n(C^{IN}_{va(n,t')n}y^{NB}_{va(n,t')} + C^{SH}_{va(n,t')n}y^{SH}_{va(n,t')})\\
&+\sum_{v\in\mathcal{V}_{\bar{T}}}(\sum_{n\in\mathcal{L}_{\bar{T}}} P_nR^{SV}_{vn}y^P_{vn}
\end{aligned}$$

$$- \sum_{t' \in \mathcal{T}} \sum_{t=\bar{T}}^{t'+M} \sum_{n \in \mathcal{L}_t} P_n (C^{IN}_{va(n,t')n} y^{NB}_{va(n,t')} + C^{SH}_{va(n,t')n} y^{SH}_{va(n,t')}))$$
$$+ \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{L}_t} \sum_{v \in \mathcal{V}_t} P_n (R^{SC}_{vn} y^{SC}_{vn} + R^{SE}_{vn} y^{SE}_{vn})$$

Demand constraints

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_{vk} x_{vrn} + n_{ikn} \geq D_{ikn}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^C_t, k \in \mathcal{K}, n \in \mathcal{L}_t,$$
$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_{vk} x_{vrn} \geq D_{ikn} \delta_{in}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^O_t, k \in \mathcal{K}, n \in \mathcal{L}_t,$$

Capacity constraints

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_v x_{vrn} + \sum_{k \in \mathcal{K}} n_{ikn} \geq \sum_{k \in \mathcal{K}} D_{ikn}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^C_t, n \in \mathcal{L}_t,$$
$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} Q_v x_{vrn} \geq \sum_{k \in \mathcal{K}} D_{ikn} \delta_{in}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^O_t, n \in \mathcal{L}_t,$$

Frequency constraints

$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} x_{vrn} \geq F_{in}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^C_t, n \in \mathcal{L}_t,$$
$$\sum_{v \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_{ivt}} x_{vrn} \geq F_{in} \delta_{in}, \qquad t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}^O_t, n \in \mathcal{L}_t,$$

Time constraints

$$\sum_{r \in \mathcal{R}_{vt}} Z_{vr} x_{vrn} \leq Z_v (y^P_{vn} + h^I_{vn} - h^O_{vn} - l_{vn}), \qquad t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t,$$

Optional trades constraints

$$\delta_{ia(n)} \leq \delta_{in}, \qquad t \in \mathcal{T} \setminus \{0, 1\}, i \in \mathcal{N}^O_t, n \in \mathcal{L}_t,$$

Pool constraints

$$y^P_{v0} = Y^{IP}_v \qquad v \in \mathcal{V}_0$$

$$
\begin{aligned}
y^P_{vn} &= y^P_{v,a(n)} - y^{SC}_{v,a(n)} + y^{SH}_{v,a(n)} - y^{SE}_{v,a(n)} && t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t \setminus \mathcal{V}^N_t, n \in \mathcal{L}_t, \\
y^P_{vn} &= y^{NB}_{va(n,t-\bar{T}_v{}^L)} && t \in \mathcal{T} : t \geq \bar{T}_v{}^L, v \in \mathcal{V}^N_t, n \in \mathcal{L}_t, \\
y^P_{vn} &= Y^{NB}_{vn} && t \in \mathcal{T} : t < \bar{T}_v{}^L, v \in \mathcal{V}^N_t, n \in \mathcal{L}_t, \\
y^P_{vn} &\geq l_{vn} - h^I_{vn} + h^O_{vn} && t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \\
y^P_{vn} &= y^{SC}_{vn} && t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t \setminus \mathcal{V}_{t+1}, n \in \mathcal{L}_t,
\end{aligned}
$$

Charter in constraints

$$
\begin{aligned}
y^{SH}_{vn} &\leq \overline{SH}_{vn}, && t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \\
y^{SE}_{vn} &\leq \overline{SE}_{vn}, && t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \\
h^I_{vn} &\leq \overline{CI}_{vn}, && t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \\
h^O_{vn} &\leq \overline{CO}_{vn}, && t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \\
\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} y^{SH}_{vn} &\leq \overline{SH}_n, && t \in \mathcal{T} \setminus \{\bar{T}\}, n \in \mathcal{L}_t, \\
\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} y^{SE}_{vn} &\leq \overline{SE}_n, && t \in \mathcal{T} \setminus \{\bar{T}\}, n \in \mathcal{L}_t, \\
\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} h^I_{vn} &\leq \overline{CI}_n, && t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t, \\
\sum_{v \in \mathcal{V}_t \setminus \mathcal{V}^N_t} h^O_{vn} &\leq \overline{CO}_n, && t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t,
\end{aligned}
$$

Cash flow constraints

$$
\begin{aligned}
\sum_{v \in \mathcal{V}_0} (R^{SE}_{v0} y^{SE}_{v0} + R^{SC}_{v0} y^{SC}_{v0}) - \sum_{v \in \mathcal{V}^{IN}_0} (C^{IN}_{v00} y^{NB}_{v0} + C^{SH}_{v00} y^{SH}_{v0}) + B \geq \bar{F}, \\
f^I_n - f^O_n \geq \bar{F}, \qquad t \in \mathcal{T} \setminus \{0\}, n \in \mathcal{L}_t,
\end{aligned}
$$

Bounds on the decision variables

$$
\begin{aligned}
y^{NB}_{vn} &\in \mathbb{Z}^+, && t \in \mathcal{T} : t \leq \bar{T} - \bar{T}_v{}^L, v \in \mathcal{V}^N_{t+T^L}, n \in \mathcal{L}_t, \\
y^{SC}_{vn}, y^{SH}_{vn}, y^{SE}_{vn} &\in \mathbb{Z}^+, && t \in \mathcal{T} \setminus \{\bar{T}\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \\
y^P_{vn} &\in \mathbb{R}^+, && t \in \mathcal{T}, v \in \mathcal{V}_t, n \in \mathcal{L}_t, \\
h^I_{vn}, h^O_{vn}, l_{vn} &\in \mathbb{R}^+, && t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, n \in \mathcal{L}_t,
\end{aligned}
$$

$$
\begin{aligned}
x_{vrn} &\in \mathbb{R}^+, && t \in \mathcal{T} \setminus \{0\}, v \in \mathcal{V}_t, r \in \mathcal{R}_{vt}, n \in \mathcal{L}_t, \\
n_{ikn} &\in \mathbb{R}^+, && t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^C, k \in \mathcal{K}, n \in \mathcal{L}_t \\
\delta_{in} &\in \{0,1\}, && t \in \mathcal{T} \setminus \{0\}, i \in \mathcal{N}_t^O, n \in \mathcal{L}_t,
\end{aligned}
$$

Conditional Value-at-Risk model

Sets	
$\mathcal{T}^F$	The set of periods in the first stage
$\mathcal{T}^S$	The set of periods under uncertainty, i.e. all periods after the first stage
Parameters	
α	Confidence level
$\bar{F}_\alpha$	The minimum expected cash flow allowed under confidence level α
Variables	
ζ	Artificial variable for CVaR constraints
η_n	Artificial variable for CVaR constraints at node n

Hard cash flow constraints

$$
\begin{aligned}
\sum_{v \in \mathcal{V}_0} (R_{v0}^{SE} y_{v0}^{SE} + R_{v0}^{SC} y_{v0}^{SC}) - \sum_{v \in \mathcal{V}_0^{IN}} (C_{v00}^{IN} y_{v0}^{NB} + C_{v00}^{SH} y_{v0}^{SH}) + B \geq \bar{F}_\alpha, \\
f_n^I - f_n^O \geq \bar{F}_\alpha, \qquad t \in \mathcal{T}^F \setminus \{0\}, n \in \mathcal{L}_t,
\end{aligned}
$$

CVaR constraints

$$
\begin{aligned}
\zeta + \frac{1}{1-\alpha} \sum_{n \in \mathcal{L}_t} P_n \eta_n \geq \bar{F}_\alpha, && t \in \mathcal{T}^S, \\
\eta_n \leq f_n^I - f_n^O - \zeta, && t \in \mathcal{T}^S, n \in \mathcal{L}_t, \\
\eta_n \in \mathbb{R}^-, && t \in \mathcal{T}^S, n \in \mathcal{L}_t
\end{aligned}
$$