Uncertainty in fleet renewal: a case from maritime transportation

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Abstract

This paper addresses the fleet renewal problem and particularly the treatment of uncertainty in the maritime case. A stochastic programming model for the maritime fleet renewal problem is presented. The main contribution is that of assessing whether or not better decisions can be achieved by using stochastic programming rather than employing a deterministic model and using average data. Elements increasing the relevance of uncertainty are also investigated. Tests performed on the case of Wallenius Wilhelmsen Logistics, a major liner shipping company, show that solutions to the model we present perform noticeably better than solutions obtained using average values.

1 Introduction

Renewing a fleet of vehicles is one of the important strategic decisions in most transportation contexts. The ageing of vehicles and the development of new and more efficient technologies force decision makers to decide whether and when to replace them. However, the problem is not only related to the replacement of old vehicles. Due to market changes, the transportation provider's goal is also to adapt the fleet capacity and characteristics to new market requirements. Therefore, the task of renewing a transportation fleet consists of choosing the number and types of vehicles to add to the fleet or dispose of in order to efficiently cope with customers' demand. In addition, decisions about the timing and the type of renewing actions (e.g. purchase or charter of additional vehicles) must be made. We will refer to this problem as the *fleet renewal problem* (FRP).

FRPs are particularly crucial in maritime transportation. We refer to the maritime counterpart of the FRP as the *maritime fleet renewal problem* (MFRP). The long lifetime of ships and the uncertainty in demand and freight rates, which characterize the maritime industry, increase the importance of good fleet renewal plans. Fleets of ships are typically heterogeneous, at least in technological characteristics (e.g. dimension, capacity, speed, operating constraints) and costs. Kavussanos and Visvikis (2006) describe shipping markets as capital intensive, cyclic, volatile, seasonal and exposed to the international business environment. Furthermore, tonnage imbalances are common. The supply of tonnage reacts slowly in response to market changes, especially because of the long lead time in the delivery of new ships. As a consequence, when demands (and freight rates) are high, shipping companies typically rush to acquire second-hand ships, which are delivered faster than new ones, but whose prices quickly rise accordingly. When used ships become too expensive, new buildings are ordered. In the meantime the demand for maritime transportation may fall and when new buildings are delivered, oversupplies may result and freight rates decrease. An example of this dates back to 2009 when, despite the fact that the total world seaborne trade volume fell by 4.5% from

2008, the total world fleet increased by 7% due to deliveries of ships ordered prior to the downturn. The resulting oversupply of tonnage led to a surge of over 300% in demolitions of old ships (UNCTAD 2010).

This paper presents a stochastic programming model for the MFRP which takes into account the uncertainty affecting the shipping market. The formulation we propose for the maritime case can easily be adapted to more general cases. We apply it to the case of *Wallenius Wilhelmsen Logistics*, a major shipping company engaged in the transportation of rolling equipment. Our main contribution is to analyze whether or not better decisions can be made by using stochastic programming rather than averaging the uncertain data and employing a deterministic model, and investigate the circumstances that increase the role of uncertainty. We first give a brief literature overview in Section 2. In Section 3 we describe the general MFRP whereas in Section 4 we present the stochastic programming model for the problem. Section 5 introduces the case of WWL and in Section 6 we show and analyze the results of the computational study. Conclusions are drawn in Section 7.

2 Literature review

The MFRP has received very little attention in the research literature, as reported by Pantuso et al. (2014). Xie et al. (2000) and Meng and Wang (2011) used dynamic programming to model the transition of a fleet from one year to another. In both cases the decision was based on the underlying fleet deployment. Cho and Perakis (1996) studied the expansion of a shipping fleet by evaluating a set of ships to buy, build or charter. Alvarez et al. (2011) presented a method to find a robust renewal of the fleet, protecting against an uncertain evolution of the shipping market.

More research has focused on the *fleet size and mix problem* (FSMP) and on the *fleet size and mix vehicle routing problem* (FSMVRP). The former consists of choosing the number and types of vehicles to use in order to perform a given transportation task. The latter combines the selection of the number and types of vehicles with the underlying vehicle routing operations. Both the FSMP and the FSMVRP can be seen as static variants of the FRP, where the given transportation demand is assumed to remain unchanged as time goes by (e.g. typically only one time period is considered). The FRP emphasizes instead the evolution of the fleet in response to market changes and, unlike the FSMP and the FSMVRP, implies that an existing fleet adapts from period to period.

A comprehensive literature survey on the FSMP, and its variants, can be found in Hoff et al. (2010). The survey also discusses some application oriented papers. Most applications are on road-based problems. Examples are Avramovich et al. (1982), who presented a FSMP for vans, Wu et al. (2005), Couillard (1993), and Leung et al. (2002) who proposed decision support systems to find the fleet size and mix of trucks. More scarce is the literature on the locomotive/wagons FSMP. Exceptions are Sherali and Maguire (2000) who determined the smallest number of wagons needed to perform a given shipment of cars and Godwin et al. (2008) who described a simulation based approach to decide the locomotive fleet size and the associated deadheading policy. Within airline applications, Listes and Dekker (2005) sought robust airline fleet compositions based on the underlying aircraft allocation with uncertain passenger demand. They also stated that explicit approaches for the airline fleet composition problem have not yet been studied in the literature. Finally, Diana et al. (2006) and Loxton et al. (2012) presented papers on the general FSMP. The former focused on demand responsive transit transportation systems while the latter studied the problem of composing a fleet of vehicles where the number of vehicles required in every period is stochastic.

Maritime applications have received little attention compared to road-based ones. Furthermore, most FSMVRPs do not necessarily match maritime problems which typically do not have a vehicle routing structure (e.g. ships usually have different positions at the beginning of the planning horizon and no depot). Examples of maritime FSMPs are Larson et al. (1991) and Richetta and Larson (1997) who developed a simulation tool for the problem of determining the fleet of barges to move refuse to offshore dumping sites. Crary et al. (2002) faced the problem of determining the size and mix of a fleet of war ships in sight of a potential conflict with uncertain mission developments. Some papers on short-term maritime FSMPs can also be found. Examples are Meng and Wang (2010) and Halvorsen-Weare et al. (2012). In the former a container shipping company is to decide which of the available ships to use and their deployment as well as the number of charters (in or out). They proposed a chance constrained formulation to tackle demand uncertainty. The latter studied the supply service for offshore oil extraction installations with the scope of finding which vessels to operate (charter) on weekly schedules.

As far as uncertainty is concerned, only a few papers take it into account. Examples are the above mentioned papers of Meng and Wang (2010) and Loxton et al. (2012) on the FSMP, who include uncertain demand and uncertain number of vehicles needed, respectively. In contrast to these papers we focus on the the evolution of the fleet rather than its initial configuration and we consider uncertainty not only restricted to demand. Finally, Alvarez et al. (2011) considered uncertain ship values, ship prices and charter rates in a robust optimization model. We extend this to deal with uncertainty also in variable operating costs (i.e. fuel costs), scrapping values and demands.

Similar decisions are made in the *machine replacement problem* (MRP) and in the *capacity expansion problem* (CEP). The former consists of choosing when to replace any of the operating machines with new and more efficient ones. In the latter, the scope is to find the sizes of facilities to add, and optionally the timing and the location, in order to increase the total available capacity. Most papers on the MRP deal with a set of identical parallel independent machines different from each other only in age. Examples are the papers of Jones et al. (1991) and VanderVeen (1985). Hartman and Lohmann (1996) introduced the possibility of leasing machines while Chand et al. (2000) combined the MRP with the CEP assuming increasing demand and variable (i.e. increasing) number of machines. Hartman and Ban (2002) considered heterogeneous machines for a series-parallel MRP, with machines varying also in capacity. Relatively more research has been done for the CEP. Several applications are listed in the survey paper of Luss (1982). Li and Tirupati (1994) focused on expansion timing and technology selection, whereas Fong and Srinivasan (1986) dealt with timing and location. In Rocklin et al. (1984) and Rajagopalan and Soteriou (1994) the disposal of facilities generates (not necessarily positive) disposal costs. Rajagopalan et al. (1998) combined CEP and MRP deciding about how much capacity of the current technology to acquire, given uncertain future technological breakthroughs.

The MFRP is however different from both the MRP and the CEP. In the MFRP ships are added not only for replacement purposes but mainly to adapt the fleet to new market conditions. Furthermore, in the MFRP many possibilities on how to modify the fleet need to be evaluated (e.g. charters, sales, scrapping, new buildings) whereas in the MRP and CEP not many alternatives are considered (e.g. machines/facilities can only be salvaged when not efficient). Furthermore, when heterogeneous machines are considered (Hartman and Ban 2002), machines differ from each other in age and capacity. In the MFRP ships might also come with different operating constraints which characterize their deployment. Finally, MFRPs are based on the underlying traveling of the ships whereas MRPs and CEPs do not have such characteristics.

3 The maritime fleet renewal problem

The MFRP is that of deciding how many and what types of ships to add to the fleet or dispose of, as well as when and how to do so, in order to satisfy the demand at minimum cost. The main decisions regard the number and types of ships to buy, build, sell or scrap, but tactical decisions such as chartering and fleet operations have to be taken into account as well. The future value of many parameters (such as ship prices, demand and charter rates) is uncertain at the moment decisions are made, therefore decisions have to be made under uncertainty. The emphasis is however on the *here-and-now* decisions (i.e. fleet modifications) taking into account the future evolution of the market (and consequently of the fleet). Future decisions are only meant as supporting information for the here-and-now decisions. Below we provide a more thorough description of the elements of the problem.

Several alternatives are available when ships are to be added to the fleet. Ships can be built or can be bought in the second-hand market. The lead time for new buildings is typically between 15 and 36 months whereas for second hand ships the time to delivery depends mostly on their position at sea at the time of purchase. Ships can also be chartered. Stopford (2009) distinguishes three types of charter, namely bareboat charter, time charter, and voyage charter. Bareboat charter consists of hiring a ship, at an agreed rate, usually for a long period of time (years). In this case the owner of the ship pays the capital cost, while the charterer controls the ship and pays all operating expenses (e.g. crew, insurance, and fuel). Time charter consists of getting the control of the ship (including the crew) for a specified period of time paying an agreed charter rate (e.g. per day or month). The charterer bears all sailing costs (e.g. fuel and port fees), while the rest of the costs (e.g. crew and insurance) are incurred by the owner. Voyage charter consists of getting a ship to perform one (ore more) individual voyages between specified ports at an agreed rate. In this case the charterer pays a rate, generally dependent on the sailing leg and on the amount of cargo, but all other expenses (including fuel and port fees) are borne by the owner. A similar charter type is the space charter, which consists of chartering a given amount of space on board of a specific ship for a specific shipment. However, voyage charters and space charters do not modify the fleet size and mix. In the following we will refer to time charters simply as charters and will specify the type when necessary.

A number of disposal alternatives are also available. Ships can be sold in the second-hand market or scrapped. In the latter case the shipping company negotiates a scrapping revenue which is in general proportional to the weight of the steel of the ship. Furthermore, ships can be chartered out (i.e. on time charters) or be set on lay-up time, which consists of stopping the ship at a port with crew and engine activity reduced to safety levels.

Lawrence (1972) classified shipping operations into three main modes: tramp, industrial and liner shipping. Tramp shipping companies engage in *Contracts of Affreightment* (COAs) which specify a series of cargoes to be shipped between given ports, the corresponding revenues and, in many cases, time windows for pickup and delivery. Furthermore, while routing their ships, these companies try to increase their profits by carrying additional spot cargoes. Industrial shipping refers to companies which ship their own goods by means of their own controlled fleet seeking the minimization of transportation costs. Finally, liner shipping companies deploy their ships according to pre-published schedules on fixed itineraries. Schedules and itineraries influence the demand.

To meet demand, the shipping company has to deploy its ships in order to deliver the appropriate amount of cargo from origin to destination. However, the routing depends much on the transportation mode of the company. We will refer to the liner shipping case without much loss of generality. In fact, our model of the deployment consists of a high level description of voyages between geographic areas which could also represent more general cases. Alvarez et al. (2011) used a similar model to describe industrial and tramp operations. A liner shipping *trade* consists of an origin and a destination geographic area, each usually including many ports. A trade is serviced when a ship picks up cargoes at each origin port, sails to the destination area (laden sailing) and unloads the cargoes at the respective destination ports. All port calls and sailing times are fixed and published in advance. After servicing a trade the ship is ready to sail (ballast sailing) to the origin of another (or the same) trade. Figure 1 illustrates an example trade from Asia to Europe.

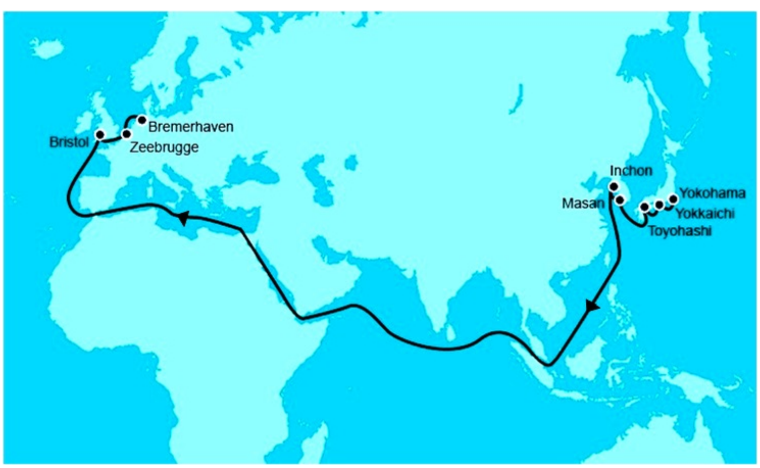

Figure 1: Example of a trade carrying goods between Northeast Asia and Europe.

The costs shipping companies incur are due to both owning and operating the fleet of ships. When ships are added to the fleet, costs for buying new or second-hand ships or chartering in ships are incurred. In the following the terms price and cost will be used interchangeably. The value of the ships depends on many factors. Adland and Koekebakker (2007) conclude that the second-hand value of a given type of ship can be described as a non-linear function of three parameters: size, age, and state of the freight market. Similarly, building prices and charter rates depend on the state of the market. Ships can be financed at least in two ways: from private funds or by loans. If ships (both new and second-hand) are financed by means of loans, the shipping company pays capital costs, i.e. the sum of principal repayments and interests. Conversely, if ships are paid with cash (coming either from cash flows or private funds) no periodic capital cost is paid. Instead, when the ship is bought in the second-hand market, there is typically only one initial expense, and when a new ship is built the payment is done at a few milestones in the construction process (e.g. signing of the contract, keel laying, and launch).

Fixed operating costs are paid for all the owned ships and consist mainly of manning, insurance, stores (i.e. consumable supplies such us domestic items used by the crew or lubricant oils for the engines), maintenance and repairs, and administrative costs. These costs typically increase with age and are incurred even if the ship does not sail. When ships are set on lay-up time, fixed operating costs are reduced due to reduced crew and stores, and engines kept on a low regime. Furthermore, if the lay-up period is long enough, the insurance premium may be renegotiated. Fixed operating costs are paid also for the ships chartered out (on time charters) but not for those chartered in (on time charters). In case of bareboat charter out, the company does not pay fixed opeparing costs. Finally, variable operating costs are incurred when a ship sails, and consist of fuel costs, port and canal fees and costs related to cargo handling at ports (e.g. loading and discharging). These costs are paid also for ships chartered in (on both time and bareboat charter), but not in case of voyage and space charters. Voyage and space charter costs are incurred when the available fleet (including time charters) is not sufficient to carry all the demand. In this case the shipping company needs to use voyage or space charters to ensure transportation of the agreed cargoes.

Revenues are generated by the remuneration for the transportation services provided, coming either from long-term contracts or from spot cargoes. Additional revenues come from selling, scrapping or chartering out ships.

When shipping companies are to renew their fleet, in most real cases only the current values of demand, new building and second-hand prices, and charter rates are known for sure. The future values of these elements are likely to be uncertain. Similarly, the future operating costs and scrapping values are also uncertain, being mainly dependent on bunker and steel prices, respectively. As far as fixed operating costs are concerned, their main driver is the crew cost whose value is still uncertain but somewhat more easily predictable and under the company's control.

4 Mathematical model

In this section a mathematical model for the MFRP described in Section 3 is presented. Modeling assumptions are discussed in Section 4.1 while the mathematical formulation is given in Section 4.2.

4.1 Modeling assumptions

In the following, we assume that ships are paid for with cash as this is common for many large shipping companies. Stopford (2009) states that most shipping businesses finance at least part of their activities from internally generated equity. Furthermore, banks rarely fully finance a ship. If this happens, this large amount of money must often be negotiated with a group of banks, which is particularly complicated when the market is poor. In addition, loans are backed by mortgages and covenants against the ships which may become too restrictive for companies with large fleets. Although new buildings are typically paid in installments made at different milestones in the building process, we assume for simplicity that their discounted sum is paid when the order is placed. Furthermore, although it is in general possible, we assume a ship cannot be bought in the second-hand market before being delivered to the company that ordered it. For simplicity we neglect the possibility of bareboat charters in and out.

We assume the negotiation of long-term contracts and the design of the shipping network to take place in a separate strategic problem. Consequently, the contracts to fulfill, the corresponding expected demands, and the origin and destination ports are input to our problem. Since the remuneration for the transportation services provided is fixed by the contracts available, we seek cost minimization. For this reason we neglect spot cargoes and the possibility of providing voyage or space charters. However, the shipping company is allowed to charter in and out ships (on time charter) as well as to pay other companies for providing voyage or space charters. In the following we will not distinguish between cargoes delivered by means of voyage charters and space charters. We will refer to the sum of these simply as voyage charters. Furthermore, it should be noticed that long-term contracts typically engage the shipping company in the transportation of a share of the customer's production. Therefore, the actual amount of cargo to ship is uncertain, as the customer's future production is not specified.

The MFRP needs to make deployment decisions in order to estimate the actual tonnage requirements. These decisions, typically made at a tactical planning level, are not meant to give any advice on the deployment itself. Furthermore, since long planning horizons are considered and market information is rarely detailed, deployment decisions are kept at an aggregate level. For an overview of the tactical ship deployment problem, see Christiansen et al. (2007).

For each trade we aggregate the demand to be shipped from its origin to its destination geographic area. Therefore, each trade can be considered as a single origin single destination route. Consider transportation of cars along the trade in Figure 1. For example, let us assume that the demands in *car equivalent units* (CEUs) are 350 000 from Inchon to Bristol, 150 000 from Yokohama to Bristol, and 100 000 from Yokohama to Bremerhaven. We associate with the trade a total aggregated demand of 600 000 CEUs and consider it as a single origin (Asia) single destination (Europe) route.

Let us introduce a complete directed graph $G = \{N, E\}$ where each vertex in N represents a trade and each edge in $E = N \times N$ represents a ballast sailing between the last and the first port of the trades it connects. It should be noticed that edges connecting vertices to themselves are allowed. With each vertex we associate the demand and service time (i.e. the sum of laden sailing and port time) of the trade it represents and with each edge we associate the duration of the corresponding ballast sailing. A small example with three trades between three geographic regions is depicted in Figure 2 while Figure 3 shows its graph representation. It should be noticed that in the graph (Figure 3) edges are drawn by dashed lines as in the map (Figure 2) as to emphasize that they represent ballast sailing and not activities. Figure 2 does not include all the ballast sailings between trades for the sake of legibility.

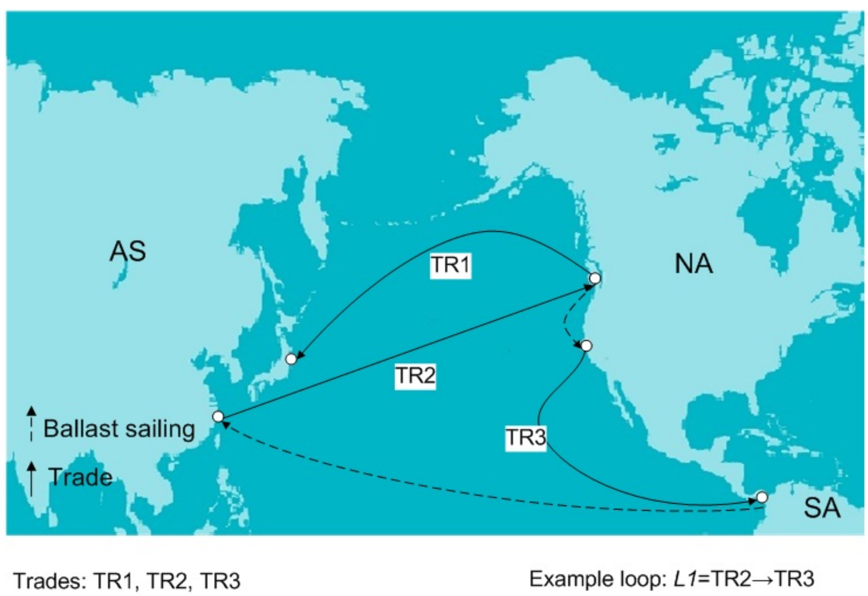

Figure 2: Example with three geographic areas, three trades and one of the possible loops.

We define a *loop* as a ship route which starts and ends in the same geographic area after servicing, in a given order, a number of trades $N' \subseteq N$. Loops of cardinality greater than two (i.e., $|N'| > 2$) represent Hamiltonian cycles over subsets $N' \subseteq N$. A ship assigned to a loop services its trades exactly once, sailing in ballast from one trade to the next, and comes back to the starting point of the first serviced trade. The ship will carry a quantity of cargo not greater than its capacity on any of the trades in the loop. Figure 2 illustrates an example loop, *L1*, involving two trades and Figure 3 its graph representation. A ship assigned to the loop *L1* services the trade TR2, sails in ballast to the origin of the trade TR3, services TR3 and sails in ballast to the origin of TR2. Analogously, if a loop has only one trade it consists of servicing the trade

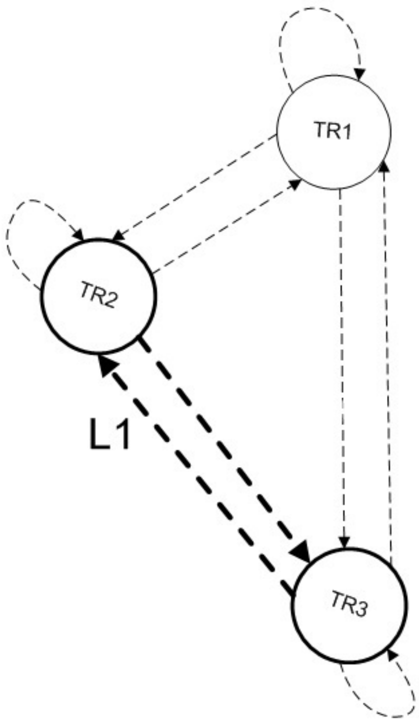

Figure 3: Graph representation of three trades and one loop. Loop L1 is drawn with thicker lines.

and sailing in ballast to its origin. Ships have to sail loops a number of times sufficient to fulfill the demand of each trade. Since in our model loops will be assigned to ship types and not to individual ships, we do not take into account the sailing between loops.

We believe these assumptions ensure a fair estimation of the tonnage requirement by balancing optimistic and pessimistic elements. On the one hand we do not take into account potential ballast sailings between loops. It is in fact possible that some ballast sailings between loops have to be made to make the routings feasible. To this extent our assumptions are optimistic since we underestimate the total sailing. On the other hand, every loop includes a ballast sailing from the last serviced trade to the starting port of the first serviced trade. If the next loop starts in a port other than the origin of the first serviced trade of the former loop, an actual ship routing decision might want the ship to sail to the origin of the next trade directly, without performing the last ballast sailing. To this extent we make a pessimistic assumption since we overestimate the sailing time.

The second-hand and charter markets consist of a finite number of operators and ships. Therefore, we assume increasing marginal ship purchase prices and charter in rates and decreasing marginal ship selling prices and charter out rates, i.e. ships become more/less expensive when the competition increases/decreases. In order to keep the model linear, second-hand costs, selling prices and charter rates are described by piecewise constant functions. The piecewise constant function is created by means of *fares*. Each fare is characterized by a price (or equivalently charter rate) and the number of ships available at that fare. When the company has already purchased/chartered in all the ships available at a given fare, the next ship must be purchased/chartered in at the next, more expensive, fare. Similarly, when the shipping company has already sold/chartered out all the ship which can be sold/chartered out at a given fare, the next ship must be sold/chartered out at the following, cheaper fare. Figure 4 gives a qualitative description of the fares for second-hand prices (Figure 4a) and charter out rates (Figure 4b). As an example, if the shipping company needs four ships, they can buy three ships at price $p1$, but must buy the fourth at price $p2$. We assume that, within the same period, selling prices are always lower than purchasing prices (at any fare) for the same ship due to transaction costs. Similarly, scrapping values are assumed lower than purchasing prices but can be higher than selling prices.

(a) Second-hand purchase price fares

(b) Charter out fares

Figure 4: Qualitative fares description.

Finally, we define a time period (a period in the following) as any interval of time in which decisions can be made or implemented. We assume fleet renewal plans are made at the end of any period. For this reason, second-hand ships bought become available in the next period. Similarly, ships sold or scrapped are delivered to the buying company or ship breaker, respectively, in the next period. We define a *stage* as any point in time at which new information is revealed. Future values of the uncertain elements are discretized in order to model the whole random process with a scenario tree, where scenarios are complete realizations of all the uncertain elements over the planning horizon. Since the company has a relatively high control on fixed operating costs, we assume them (as well as the lay-up savings) to be deterministic for the whole planning horizon.

4.2 Formulation

Let $T = \{0, \dots, \bar{T}\}$ be the set of periods, indexed by t. Ships can be bought in the second-hand market, sold or scrapped in $t \in T \setminus \{\bar{T}\}$ as they will be included in or excluded from the fleet in the following period. Ships can be built in $t \in T : t \leq \bar{T} - \bar{T}_v^L$, where $\bar{T}_v^L$ is the number of periods of lead time for building a ship of type v. Ships can be operated (i.e. deployed), chartered (in or out) or set on lay-up time in $t \in T \setminus \{0\}$. Notice that deployment, charter and lay-up decisions for $t = 0$ are not taken into account as they are not influenced by current fleet renewal decisions, nor do they influence future tonnage requirements. Let T^S be the set of periods in which new buildings ordered before the beginning of the planning horizon can be delivered. Let S be the set of scenarios, indexed by s.

Let V_t be the set of ship types existing in the market in period t, indexed by v. Ship types differ from each other in speed, capacity, fuel consumption and age. Let A_{vt} be the age of a ship of type v in period t and $\bar{A}$ the lifetime of a generic ship. The lifetime of a ship typically does not depend on the type of ship but on the company's policy and is between 20 and 30 years. Ship types belong to V_t as long as $A_{vt} \geq 0$ and, possibly, as long as $A_{vt} \leq \bar{A}$ if the company's policy is to strictly remove ships from operations when they reach their lifetime. Then let $V_t^N = \{v \in V_t : A_{vt} = 0\}$ be the set of new ship types in period t, i.e. the new ship types that can be delivered by the ship builder. Then let $F^{SH}, F^{SE}, F^{CI}, F^{CO}$, indexed by f, be the sets of second-hand, selling, charter in and charter out fares,respectively.

Let N_t be the set of trades operated in period t indexed by i, R_t the set of loops in period t, $R_{vt} \subseteq R_t$ the set of loops which can be sailed by a ship of type v in period t and $R_{ivt} \subseteq R_{vt}$ the set of loops servicing trade i which can be sailed in period t by a ship of type v, all indexed by r.

With respect to the decision variables, for ships of type v in period t, under scenario s, let y_{vts}^{NB} be the number of new buildings ordered, y_{fvts}^{SH} and y_{fvts}^{SE} the number of ships bought and sold at fare f in the second-hand market, respectively, and y_{vts}^{SC} the number of ships scrapped. Then let h_{fvts}^{I} and h_{fvts}^{O} be the number of ships chartered in and out for one period at fare f, respectively, where fractions indicate the portion of the period the ship has been chartered for, e.g. 2.5 indicates the charter of two ships for one period and one ship for half of a period. Similarly, let l_{vts} be the number of ships on lay-up for one period, where fractions indicate the portion of the period ships have been on lay-up. Let y_{vts}^{P} be the number of ships in the pool and x_{vrts} the number of loops r performed. Finally, let s_{its} be the amount of cargo delivered in period t by voyage charters on trade i under scenario s.

The MFRP is hence:

$$\min \quad z = \sum_{s\in S} p_s \left\{ \sum_{t\in T: t\leq \bar{T}-\bar{T}^L_v} \sum_{v\in V^N_{t+\bar{T}^L_v}} C^{NB}_{vts} y^{NB}_{vts} \right. \tag{1a}$$

$$+ \sum_{t\in T\setminus\{\bar{T}\}} \sum_{v\in V_t} \left(\sum_{f\in F^{SH}} C^{SH}_{fvts} y^{SH}_{fvts} \right. \tag{1b}$$

$$\left. - \sum_{f\in F^{SE}} R^{SE}_{fvts} y^{SE}_{fvts} - R^{SC}_{vts} y^{SC}_{vts} \right) \tag{1c}$$

$$+ \sum_{t\in T\setminus\{0\}} \sum_{v\in V_t} \left(\sum_{f\in F^{CI}} C^{CI}_{fvts} h^I_{fvts} - \sum_{f\in F^{CO}} R^{CO}_{fvts} h^O_{fvts} \right. \tag{1d}$$

$$\left. + C^{OP}_{vt} y^P_{vts} - R^{LU}_{vt} l_{vts} + \sum_{r\in R_{vt}} C^{TR}_{vrts} x_{vrts} \right) \tag{1e}$$

$$\left. + \sum_{t\in T\setminus\{0\}} \sum_{i\in N_t} C^{VO}_{its} s_{its} - \sum_{v\in V_{\bar{T}}} R^{SV}_{vs} y^P_{v\bar{T}s} \right\} \tag{1f}$$

The objective function (1a)-(1f) represents the expected total cost of providing and operating ships within the planning horizon, where p_s is the probability of scenario s taking place. Expression (1a) represents the total expected cost of building new ships where C^{NB}_{vts} is the building cost of a ship of type v in period t under scenario s. Expression (1b) represents the expected cost for buying second-hand ships where C^{SH}_{fvts} is the cost of a ship of type v in period t at fare f under scenario s in the second-hand market. Expression (1c) represents the expected revenues for selling and scrapping ships, where, for a ship v, in period t under scenario s, R^{SE}_{fvts} and R^{SC}_{vts} are the selling price at fare f and the scrapping value, respectively. Expression (1d) sums up the expected costs and revenues of chartering in and out ships, respectively, where C^{CI}_{fvts} and C^{CO}_{fvts} are the charter in cost and revenue for one period at fare f, for ship type v, in period t and scenario s, respectively. Expression (1e) represents the sum of expected fixed operating costs, lay-up savings and variable operating costs where, for ships of type v in period t under scenario s, C^{OP}_{vt} is the fixed operating cost for one period, R^{LU}_{vts} is the lay-up saving for one period and C^{TR}_{vrts} is the cost of performing a loop r. It should be noticed that the fixed operating costs and lay-up savings do not vary with the scenarios. Finally, expression (1f) represents the sum of expected voyage charter expenses and value of the ships in the pool at the end of the planning horizon (i.e. the sunset value, see Alvarez et al. (2011)), where C^{VO}_{its} is the voyage charter cost for one unit of cargo on trade i in period t and scenario s, and R^{SV}_{vs} is the value of a ship of type v in period $\bar{T}$ under scenario s. All the monetary quantities are appropriately discounted.

The objective function could match the case of industrial shipping without modifications. In case of tramp shipping revenue terms should be included in the objective function as well as changing it to a profit maximization function (see for example Alvarez et al. (2011)).

The problem is subject to constraints (2)-(23).

$$y^P_{vts} = y^P_{v,t-1,s} + \sum_{f\in F^{SH}} y^{SH}_{f,v,t-1,s} - \sum_{f\in F^{SE}} y^{SE}_{f,v,t-1,s} - y^{SC}_{v,t-1,s} \qquad t\in T\setminus\{0\}, v\in V_t\setminus V^N_t, s\in S \tag{2}$$

$$y^P_{vts} = y^{NB}_{v,t-\bar{T}^L_v,s} \qquad t\in T: t\geq \bar{T}^L_v, v\in V^N_t, s\in S \tag{3}$$

$$y^P_{vts} = Y^{NB}_{vt} \qquad t\in T^S\setminus\{0\}, v\in V^N_t, s\in S \tag{4}$$

$$y^P_{v0s} = Y^P_v \qquad v\in V_0, s\in S \tag{5}$$

$$\sum_{f\in F^{SE}} y^{SE}_{fvts} + y^{SC}_{vts} \leq y^P_{vts} \qquad t\in T\setminus\{\bar{T}\}, v\in V_t: A_{vt} = \bar{A}, s\in S \tag{6}$$

Constraints (2)-(4) keep the balance between ships joining and leaving the fleet. Constraints (2) refer to ships bought or sold in the second-hand market or scrapped while constraints (3) refer to new building orders and deliveries whereas constraints (4) keep track of the deliveries of new buildings ordered in the sunk periods, where Y^{NB}_{vt} is the number of new ships of type v delivered in period t. Constraints (5) define the initial pool of ships. Finally, if the company's policy is to strictly remove ships from operation (i.e. from sets V_t) at a given age, constraints (6) ensure consistent selling or scrapping.

$$\sum_{v\in V_t}\sum_{r\in R_{ivt}} \bar{Q}_v x_{vrts} + s_{its} \geq D_{its} \qquad t\in T\setminus\{0\}, i\in N_t, s\in S \qquad (7)$$

$$\sum_{r\in R_{vt}} Z_{rv} x_{vrts} \leq Z_v(y^P_{vts} + \sum_{f\in F^{CI}} h^I_{fvts} - \sum_{f\in F^{CO}} h^O_{fvts} - l_{vts}) \qquad t\in T\setminus\{0\}, v\in V_t, s\in S \qquad (8)$$

$$\sum_{f\in F^{CO}} h^O_{fvts} + l_{vts} \leq y^P_{vts} \qquad t\in T\setminus\{0\}, v\in V_t, s\in S \qquad (9)$$

Constraints (7) ensure the satisfaction of the demand on each trade. The demand on a trade can be satisfied by sailing loops servicing the trade with owned or chartered ships or by means of voyage charters, where $\bar{Q}_v$ is the total capacity of ship type v and D_{its} is the demand on trade i in period t under scenario s. Constraints (8) ensure that the sailing time of each type of ship is consistent with the number of ships of that type operated, where Z_{rv} is the time a ship of type v needs to perform a loop r, and Z_v is the total available time in one period for a ship of type v. Constraints (9) ensure that the ships set on lay-up or chartered out are actually available.

$$h^I_{fvts} \leq L^{CI}_{fvt} \qquad f\in F^{CI}, t\in T\setminus\{0\}, v\in V_t, s\in S \qquad (10)$$

$$h^O_{fvts} \leq L^{CO}_{fvt} \qquad f\in F^{CO}, t\in T\setminus\{0\}, v\in V_t, s\in S \qquad (11)$$

$$y^{SH}_{fvts} \leq L^{SH}_{fvt} \qquad f\in F^{SH}, t\in T\setminus\{\bar{T}\}, v\in V_t, s\in S \qquad (12)$$

$$y^{SE}_{fvts} \leq L^{SE}_{fvt} \qquad f\in F^{SE}, t\in T\setminus\{\bar{T}\}, v\in V_t, s\in S \qquad (13)$$

Constraints (10)-(11) set the limit to the number of ships that can be chartered in and out, respectively, at each fare. L^{CI}_{fvt} and L^{CO}_{fvt} represent the number of ships of type v available at fare f in in period t to charter in and out, respectively. Similarly, constraints (12)-(13) set the limits to the number of ships that can be bought and sold at each fare in the second-hand market, respectively, where L^{SH}_{fvt} and L^{SE}_{fvt} are the respective thresholds.

$$y^{NB}_{vts} \in \mathbb{Z}^+ \qquad t\in T: t\leq \bar{T}-\bar{T}^L_v, v\in V^N_{t+\bar{T}^L_v}, s\in S \qquad (14)$$

$$y^{SH}_{fvts} \in \mathbb{Z}^+ \qquad f\in F^{SH}, t\in T\setminus\{\bar{T}\}, v\in V_t, s\in S \qquad (15)$$

$$y^{SE}_{fvts} \in \mathbb{Z}^+ \qquad f\in F^{SE}, t\in T\setminus\{\bar{T}\}, v\in V_t, s\in S \qquad (16)$$

$$y^{SC}_{vts} \in \mathbb{Z}^+ \qquad t\in T\setminus\{\bar{T}\}, v\in V_t, s\in S \qquad (17)$$

$$y^{P}_{vts} \in \mathbb{R}^+ \qquad t\in T, v\in V_t, s\in S \qquad (18)$$

$$x_{vrts} \in \mathbb{R}^+ \qquad t\in T\setminus\{0\}, v\in V_t, r\in R_{vt}, s\in S \qquad (19)$$

$$h^I_{fvts} \in \mathbb{R}^+ \qquad f\in F^{CI}, t\in T\setminus\{0\}, v\in V_t, s\in S \qquad (20)$$

$$h^O_{fvts} \in \mathbb{R}^+ \qquad f\in F^{CO}, t\in T\setminus\{0\}, v\in V_t, s\in S \qquad (21)$$

$$l_{vts} \in \mathbb{R}^+ \qquad t\in T\setminus\{0\}, v\in V_t, s\in S \qquad (22)$$

$$s_{its} \in \mathbb{R}^+ \qquad t\in T\setminus\{0\}, i\in N_t, s\in S \qquad (23)$$

Constraints (14)-(17) impose non-negativity and integrality on the respective variables, constraints (18)-(23) restrict the related variables to real and non-negative values.

Finally, it should be noticed that the MFRP can model both two-stage and multistage configurations, depending on the way scenarios are generated (i.e. the number of times new information is obtained). As an example, if new information is revealed at only one point in time (during the planning horizon) we will have a two-stage program whereas if new information is delivered at more times we will have a multistage program.

The model is to be considered nonanticipative, but nonanticipativity constraints are not shown for the sake of simplicity. Nonanticipativity constraints ensure that decisions are made based only on information available at the moment the decisions are made, thus not anticipating the future. We stress however that future decisions are not meant to be implemented. They provide the right background to make first-stage decisions. First-stage decisions are to be understood as a suggestion on what to do today, based on the uncertain future.

5 Case study

In this section we introduce a case study based on Wallenius Wilhelmsen Logistics (WWL), a major liner shipping company engaged in overseas transportation of cars, high and heavy vehicles and breakbulk cargo. They currently operate a fleet of more than 60 specialized ships by which they carry cargo to many ports around the world. By the end of every year WWL update their long-term strategy. Among other strategic issues they decide how to modify the current fleet which includes decisions on how many and which types of ships to acquire or dispose of and how to do so. This corresponds to solving the MFRP modeled in Section 4. WWL typically stipulate long-term contracts with customers, which commit the company to carry a share of the customers' production. However, the future customers' production is not specified in the contract and is therefore uncertain and so is the future market situation (e.g. ship prices and charter rates). Thus, WWL base their strategic planning on five year forecasts provided by their market intelligence department. These forecasts typically bring information on the development of geographic areas and products, and hardly consider information relative to individual customers. The market intelligence department gives WWL a preliminary view on the future. As in the MFRP proposed, we can consider as uncertain elements: building prices, second-hand prices, selling prices, charter rates (in and out), sunset values, scrapping values, variable operating costs and demands.

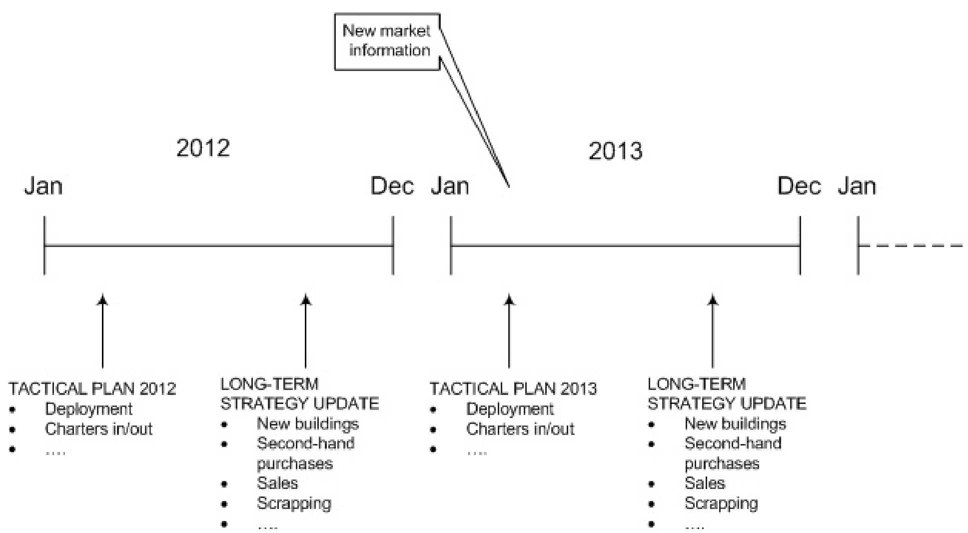

Figure 5: Decisions sequence.

In addition to the long-term strategy, every year WWL perform some tactical planning which mainly consists of decisions on the deployment of the ships on the operated trades and on the number of charters in or out. Figure 5 illustrates the sequence of decisions. In the following we describe in detail the types of cargo transported (Section 5.1), the ships used in the fleet (Section 5.2), the trades WWL operate (Section 5.3) and finally, we introduce new constraints needed to fit the model (1a)-(23) to the WWL case (Section 5.4).

5.1 Products

WWL transport three types of products: cars, high and heavy vehicles and breakbulk cargo. High and heavy vehicles (HH in the following) include trucks and out of gauge vehicles such as agricultural vehicles, construction vehicles and cranes. Breakbulk products (BB in the following) are non-rolling and non-containerized goods that must be individually lifted or placed on trailers to be boarded (e.g. big turbines, train coaches, boats).

The standard measurement unit for cars cargo is RT43, based on the dimensions of a Toyota Corolla 1967 model, where 1 RT43 is equal to 9.1 cbm. As an example a Toyota Corolla, 2005 model, has a volume of 11.4 cbm, equivalent to 1.25 RT43. This measurement unit is also used for HHs and BBs in order to have a homogeneous measurement system.

5.2 Ships

WWL operate vessels belonging to three main families: *pure car truck carrier* (PCTC), *large car truck carrier* (LCTC) and *roll-on/roll-of carrier* (RORO) — see Figure 6. PCTCs are optimized for cars and trucks as they have fixed car decks (up to 13) occupying more than 80% of the total volume capacity (up to 6500 RT43). They also have hoistable decks that allow the ship to take some taller BBs. LCTCs have bigger capacity and more hoistable and strengthened decks (up to six) and strengthened ramps that make them more flexible commercially. The height of the decks can in fact be adjusted according to the height of the cargo on board, and strengthened decks and ramps allow for heavier units of HHs and BBs. The allowance of HHs and BBs is therefore higher than in PCTCs. Finally, ROROs are optimized for HHs and BBs with cars as complementary cargo. They represent the most flexible alternative as they can adapt to any size of HH and BB cargo but can also be filled by cars if needed.

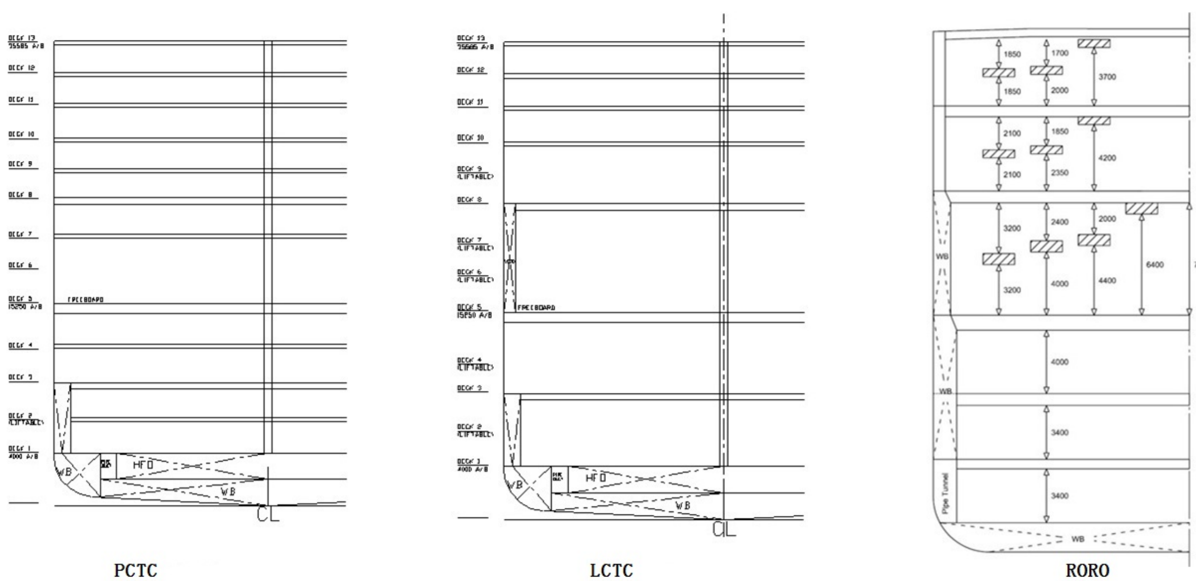

Figure 6: Cross sections showing the different space distribution between different families of ships.

Every family of ships can carry all types of cargo but in different proportions. Table 1 shows the capacities of three of the ships in the current fleet. It should be noticed that Ship3 is more flexible than Ship2, which in turn is more flexible than Ship1. While Ship3 can carry almost as many cars as Ship1, Ship1 cannot be converted to carry as many HHs and BBs as Ship3. As an example, a possible feasible load pattern for Ship1 would be 5000 RT43 of cars, 1100 RT43 of HHs and 164 of BBs. It respects the total capacity, the individual product capacities and the maximum allowance for non-car cargoes. An infeasible pattern would be 4046 RT43 of cars, 2000 RT43 of HHs and 200 of BBs, because it violates the total amount of non-car cargoes allowable.

Table 1: Capacities of three of the available ships.

	Capacities [RT43]					
Ship	Total	HH+BB	Car	HH	BB	Family
Ship1	6 246	2 100	6 246	2 100	600	PCTC
Ship2	7 934	3 600	7 934	3 600	1 600	LCTC
Ship3	7 500	7 500	6 000	7 500	4 100	RORO

5.3 Trades

WWL operate several trades worldwide. Figure 7 shows four of these. The duration of each trade may in general vary from one service to another as not all ports on the trade are called in all voyages. However, since the range of variation is usually small we can approximate it by assigning each trade an average duration. Furthermore, WWL may want to control the number of times trades are serviced each year, setting a lower bound on the service frequency of the trades. This may apply because of strategic policies or contract requirements.

Figure 7: Example of trades operated by WWL (from www.2wglobal.com).

5.4 Additional constraints

In order to make model (1a)-(23) suitable for the WWL case we need to replace capacity constraints (7) with constraints (24)-(27). Let us define P indexed by p as the set of product types transported by WWL.

$$q_{pvrts} \leq \bar{Q}_{vp} x_{vrts} \qquad t \in T \setminus \{0\}, v \in V_t, r \in R_{vt}, p \in P, s \in S \qquad (24)$$

$$\sum_{p \in P \setminus \{\text{car}\}} q_{pvrts} \leq \bar{Q}^{NC}_v x_{vrts} \qquad t \in T \setminus \{0\}, v \in V_t, r \in R_{vt}, s \in S \qquad (25)$$

$$\sum_{p \in P} q_{pvrts} \leq \bar{Q}_v x_{vrts} \qquad t \in T \setminus \{0\}, v \in V_t, r \in R_{vt}, s \in S \qquad (26)$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} q_{pvrts} + s_{pits} \geq D_{pits} \qquad t \in T \setminus \{0\}, i \in N_t, p \in P, s \in S \qquad (27)$$

Constraints (24) state that the amount of cargo of type p, carried with ship type v on loop r in period t under scenario s, represented by q_{pvrt}, must not be greater than the amount of cargo p ships of type v can carry in all the voyages performed on loop r, where $\bar{Q}_{vp}$ represents the capacity of cargo p for ship type v. Constraints (25) state that the number of voyages made on each loop has to be enough to carry the necessary amount of products other than cars, where $\bar{Q}^{NC}_v$ is the capacity of non-car cargoes on ship type v. Constraints (26) ensure that the total ship capacity is respected, where $\bar{Q}_v$ is the total capacity of ship type v. Finally, constraints (27) ensure that the amount of any product carried is enough to satisfy the demand of that product on each trade, where D_{pits} is the demand of product p on trade i in period t under scenario s and s_{pits} is the amount of product p sent by means of voyage charters on trade i in period t under scenario s.

As WWL operate in a specialized market, with a small number of competitors and ships, the number of ships available on time charters is limited. Constraints (28) are added to limit the number of charters, where L^{CI} is the maximum number of ships available for a one year charter. Constraints (29) are added to impose service frequency on the trades, where $N^C_t \subseteq N_t$ is the set of trades with frequency imposed upon them (*controlled trades* in the following) and H_{it} is the minimum number of times trade i is to be serviced in period t. Furthermore, some of WWL's ships are built according to their requirements (e.g. some new RORO) and are therefore not available in the second-hand market. Hence, they cannot be chartered in, but can only be built. We will refer to them as *special ships*. Constraints (30) and constraints (31) exclude the possibility to buy or charter them in, where $V^S_t \subseteq V_t$ is the set of special ships in period t.

$$\sum_{v \in V_t} \sum_{f \in F^{CH}} h^I_{fvts} \leq L^{CI} \qquad t \in T \setminus \{0\}, s \in S \qquad (28)$$

$$\sum_{v \in V_t} \sum_{r \in R_{ivt}} x_{vrts} \geq H_{it} \qquad t \in T \setminus \{0\}, i \in N^C_t, s \in S \qquad (29)$$

$$\sum_{f \in F^{SH}} y^{SH}_{fvts} = 0 \qquad t \in T \setminus \{\bar{T}\}, v \in V^S_t, s \in S \qquad (30)$$

$$\sum_{f \in F^{CH}} h^I_{fvts} = 0 \qquad t \in T \setminus \{0\}, v \in V^S_t, s \in S \qquad (31)$$

We will refer to the resulting optimization model for the maritime fleet renewal problem for WWL as $MFRP^W$.

6 Computational study

The main scope of the computational study is to evaluate whether better decisions can be achieved through the use of the stochastic programming model proposed ($\mathrm{MFRP}^{\mathrm{W}}$) rather than using average data, which is a common approach in practice. Furthermore, we want to find out under which circumstances uncertainty matters and not. In other words, we want to discover whether randomness is important in this problem (and must therefore be appropriately included in the model) or not. In the same spirit Kall and Wallace (1994) state that even if the problem is clearly stochastic, it does not mean that we must necessarily use stochastic programming to model it and that it is extremely difficult to know if randomness matters before solving the problem and checking the results. When randomness characterizes elements of the problem, a typical approach is that of replacing the uncertain data with expected values in order to avoid the extra complexity caused by the inclusion of scenarios. The resulting model can be referred to as the *mean value problem* (MVP). To this extent Birge (1982) defined the *value of the stochastic solution* as a quantity which measures the expected return from solving a stochastic program rather than a deterministic one using expected value data (i.e. the MVP). In the computational study we measure and analyze the causes of this expected return.

All tests have been coded in Java and the problems were solved with XpressMP 7.2.1 on a HP dl165 G6, 2 x AMD Opteron 2431 2,4 GHz, 24 Gb RAM machine.

In Section 6.1 we describe the instances and the settings used to run the tests, and in Section 6.2 we describe how the random variables are discretized in order to generate scenario trees. In Section 6.3 we analyze the difference between the stochastic programming approach and its deterministic counterpart. In Section 6.4 we analyze how charter limits and frequency controls influence the results and in Section 6.5 we analyze the influence of the deployment aspects on the solutions. Finally, in Section 6.6 we discuss the complexity of the problems solved and possible ways to solve them more efficiently.

6.1 Instances description

Instances are described by the number of ship types, ns, and trades, nt, with a code ns_nt. Ship types have been created by grouping the ships in the current fleet of WWL by similar characteristics (i.e. age, speed and capacity). We use three instances, namely 6_5, 8_8 and 10_12 generated from the WWL case. Tables 2 and 3 show the ship types and the trades in the instances, respectively. Ship types and trade names are fictitious in order to preserve confidentiality. Table 2 reports capacities, age and service speed for the available ship types. Table 3 specifies the length of each trade and the frequency requirement in terms of number of services per year. As an example, instance 6_5 includes six ship types RORO1, RORO2, LCTC1, LCTC2, PCTC1 and PCTC2 whereas the other ship types are not included. The trades included in instance 6_5 are TR1, TR2, TR3, TR4 and TR5. In Table 2 the number of ships in the initial fleet has been chosen in order to make the initial fleet balanced with the market requirements. We run the MVP with different initial fleets until the difference between the number of ships bought and sold in the first stage is between minus and plus five. The set of special ship types, V_t^S, consists of ship type RORO1 for each $t = 0, \ldots, \bar{T}$ in all instances. The settings reported in what follows also apply to all instances.

Table 2: Ship types in the instances.

Ship type	Instance: 6_5	8_8	10_12	Capacity [RT43]: Car	HH	BB	Initial age [years]	Service speed [knots]
	# of ships in the initial fleet							
RORO1	0	0	0	6 000	7 500	4 100	-2 [a]	20.8
RORO2	8	9	8	4 000	5 700	3 400	22	16.5
RORO3	- [b]	-	10	4 600	6 800	3 500	8	17.0
LCTC1	0	0	0	7 600	3 600	9 00	-2	18.5
LCTC2	9	11	9	7 930	3 600	1 600	7	19.8
LCTC3	-	10	12	6 930	2 900	1 600	16	17.8
PCTC1	0	0	0	6 350	2 900	1 300	-2	18.5
PCTC2	10	12	7	6 500	1 900	800	3	18.5
PCTC3	-	13	10	6 300	2 000	500	21	16.5
PCTC4	-	-	9	6 000	2 100	400	8	16.8

[a] Negative ages, $-a$, indicate that ships of that type can be operated from period a and be ordered in period $a - \bar{T}_v^L$

[b] "-" indicates that the ship type is not considered in the instance

The length of the planning horizon is six periods (i.e. the current period and the five periods that follow). This is in accordance with the length of the market forecasts made by WWL (see Section 5). Since one period corresponds to one year, and fleet renewal decisions at WWL are made a few months before the end of the year, we assume $\bar{T}^L_v = 2, v \in V^N_t, t \in T$ (i.e. ships ordered in the current period are delivered after one period). This would correspond to around 15 months between the order placement and the delivery. We set up a two-stage stochastic program with recourse, where recourse actions are such as chartering, buying or selling, which can be made after the realization of the random variables (i.e. in the second stage). The current period (i.e. $t = 0$) represents the first stage whereas the five periods ahead belong to the second stage.

Table 3: Trades in the instances.

Trade	6_5	8_8	10_12	Length [nautical miles]	H_{it}[services/year]
TR1	x	x	x	7 500	20
TR2	x	x	x	14 500	52
TR3	x	x	x	13 500	48
TR4	x	x	x	13 000	48
TR5	x	x	x	15 021	20
TR6		x	x	19 200	48
TR7		x	x	11 700	25
TR8		x	x	10 000	48
TR9			x	7 800	20
TR10			x	7 800	48
TR11			x	4 900	48
TR12			x	8 400	48

Each uncertain element of the problem, say u_t, (e.g. the scrapping value for ship v at period t) is modeled as a random variable with expected value $E(u_t)$ uniformly distributed over a support $[E(u_t)(1 - k_{ut}), E(u_t)(1 + k_{ut})]$, where $k_{ut} \in [0, 1]$ defines the range of variation. WWL's forecasts are used as expected values. High values of k_{ut} indicate low confidence in the forecasts. Values of k_{ut} can be set, for example, by observing past accuracy of the forecasts or by analyzing the current market status. Random variables are then discretized in order to obtain scenarios as discussed in Section 6.2. Figure 8 gives a qualitative illustration of scenarios created from WWL's forecasts where the solid line represents the forecasted values and the dashed lines represent the other scenarios.

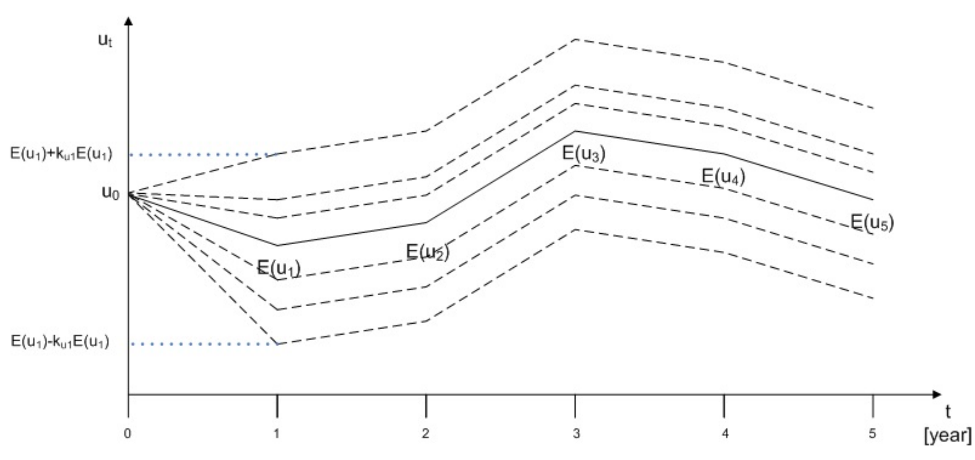

Figure 8: Qualitative description of the scenarios.

The following assumptions about the correlations between the random variables are used for all $t = 1, \ldots, \bar{T}$. For each $v \in V_t$, second-hand and selling prices at any fare, charter rates (in and out) at any fare, and sunset values (for $t = \bar{T}$) are all perfectly correlated. Similarly, for any pair $v \in V_t, v' \in V^N_{t+T^L}$, the building price of v' is perfectly correlated with second-hand prices, selling prices and charter rates (in and out) of v at any fare. In general there is some time lag between the reaction of second-hand prices and new buildings prices (see Section 1). However, the shipping segment WWL operates in is a relatively small market, and the number of ships available is limited. For this reason we are confident to assume that new building and second-hand prices react almost synchronously. Moreover, for any pair $v, v' \in V_t$, second-hand and selling prices at any fare, charter rates (in and out) at any fare, and sunset values (for $t = \bar{T}$) are all perfectly correlated. As an example, any increase in the building price of ship v' corresponds to an increase in the second-hand and selling price of ship v at all fares as well as in its charter in and charter out rates. Let us refer to these elements as *ship values*. Similarly, for any pair $v, v' \in V_t$ the scrapping values are perfectly correlated between themselves.

For any ship type $v \in V_t$ the variable operating costs on any pair of loops $r, r' \in R_{vt}$ are perfectly correlated (e.g. any increase in the variable operating cost for ship type v on loop r leads to an increase of variable operating cost also on loop r'). Similarly, the variable operating costs for any pair of ships types $v, v' \in V_t$ on loop $r \in R_t$ are perfectly correlated between themselves (e.g. any increase in the variable operating cost for ship type v on loop r leads to an increase of variable operating cost for ship type v' on loop r). Finally, given a product $p \in P$, for any pair of trades $i, i' \in N_t$ the demands of the product on the trades are perfectly correlated between themselves (i.e. an increase in the demand of p on trade i leads to an increase in its demand on all trades). This assumption can easily be modified, based on the specific case, by setting different correlations between the demand for a given product on different trades. This would allow the demand to develop independently on different trades.

For the remaining correlations we assume a strong positive correlation between variable operating costs (mainly driven by bunker prices) and demands. Bunker oil consumption increases if more sailing is needed (i.e. increase in the demand), and its price rises accordingly. Similarly, we assume strong positive correlations between demands and ship values, as demand is the main driver of ship prices and charter rates. We also assume that the demand of cars has a strong positive correlation with that of HHs and a more moderate positive correlation with that of BBs. Since they are not substitute products, an increase on one of those demands is likely to represent a general positive economic phase which would be likely to influence also the demand for other products. Finally, we assume a minor positive correlation between scrapping values and ship values and demands. Scrapping values are mainly driven by the steel price.

In Section 4.1 we defined a loop as a sequence of trades to service in a given order, coming back to the starting point. In order to prevent the evaluation of too many loops we generate only the most promising ones. Therefore, for each subset N'_t, of the set of trades N_t, we only consider as a loop the sequence of trades of shortest length. As an example, consider the trades in Figure 2 and the subset N'_t made of all the trades (i.e. $N'_t = N_t$). Six sequences (i.e. potential loops) can be made with the three trades (e.g. TR3→TR1→TR2 and TR1→TR3→TR2). If we assume that TR1→TR2→TR3 is the sequence with shortest length (i.e. least ballast sailing between the trades), it will be the one considered as a loop whilst the other five sequences will be discarded because dominated by the shortest one. This corresponds to solving an asymmetric traveling salesman problem (aTSP) for each $N'_t \subseteq N_t : |N'_t| > 2$. Let C^{MAX} be the highest cardinality $|N'_t|$ considered when generating loops.

Furthermore, the following assumptions are valid. We assume $N_1 = N_2 \dots N_{\bar{T}-1} = N_{\bar{T}}$. Similarly, for the service frequency of each trade i we assume $H_{i1} = H_{i2} \dots H_{i\bar{T}-1} = H_{i\bar{T}}$. Finally, we assume that there is no sailing restriction for the available ship types and therefore all ships can service all trades (i.e. $R_{vt} = R_t$ for any $t \in T \setminus \{0\}, v \in V_t$).

6.2 Discretization and stability

The discretization of the random variables is obtained by using a version of the scenario-generating heuristic of Høyland et al. (2003), with margins controlled by distribution functions instead of moments. Taking as input the probability distribution of each random variable and the correlations between them, it generates the desired number of scenarios for each of the random variables. In our case each scenario consists therefore of a vector of $\bar{u}$ elements containing a realization of each random variable, where $\bar{u}$ is the total number of random variables.

To make sure that the discretization method used to generate scenarios does not affect the results, we look at *in-sample stability* for stochastic programs, as discussed by Kaut and Wallace (2007). Since the chosen scenario tree generation method has random elements in it, it can produce many different scenario trees from the same data. The purpose of the test is therefore to make sure that, whatever tree we use, the objective values are approximately the same. In-sample stability was checked for each of the instances and for each setting of the problem tested in the following sections. To check the in-sample stability we measure the standard deviation of the optimal objective function value registered with different scenario trees. All settings, for all instances, showed that we had stability with $|S| = 15$ as the standard deviation did not exceed 0.6% of the average optimal objective function value in the worst case. Since the differences in the results presented in the next sections are of a higher order of magnitude, they are not much influenced by the specific scenario trees used.

6.3 Value of the stochastic solution for the MFRP$^{\mathrm{W}}$

In this section we report the results of the comparison between the MFRP$^{\mathrm{W}}$ and the corresponding MVP, i.e. a deterministic version of the MFRP$^{\mathrm{W}}$ obtained by using expected values of the uncertain elements. For each instance we measure the value of the stochastic solution (VSS) as defined in Birge (1982). This value tells how well (or how poorly) the solution to the MVP (DS – deterministic solution – in the following) performs if compared to the solution to the stochastic model (SS – stochastic solution – in the following), i.e. the difference between the expected cost for the DS and the expected cost for the SS. In order to relate the VSS to the size of the problem we measure it as a percentage of the expected cost of the SS and we refer to it as VSS%. Each test is performed by finding the VSS% averaged over 15 different scenario trees, each with $|S| = 15$. For each we set the maximum number of charters $L^{CI} = 3$, the set of controlled trades $N_t^C = \emptyset, t = 1, \ldots, \bar{T}$ and the highest loop cardinality $C^{MAX} = 2$.

Table 4 reports the value of the VSS% for all the instances. In each case the VSS% is noticeable as it may represent, for large shipping companies such as WWL, up to hundreds of million US$. The VSS% however increases with the size of the instance. This is due to the charter limit (three ships) which represents a tighter restriction in the larger instances.

Table 4: VSS% and expected costs in each instance.

	6_5	8_8	10_12
Avg VSS%	7.64	8.48	9.56
Max VSS%	8.91	9.22	10.40

Table 5 reports the solution for $t = 0$ for both the DS and the SS (for one of the scenario trees). It is important to point out that while the DS makes *here-and-now* decisions for the whole planning horizon, the SS follows the structure of the scenario tree and, beyond the decisions related to $t = 0$, nothing is decided. One difference between the two solutions is that the SS does not sell any ship in the first stage while the DS does. The SS prefers to wait until more information is revealed, whereas the DS does not capture the option of keeping ships in case they might be useful in any of the future scenarios. Kall and Wallace (1994) state: *"in a deterministic world there is never a need to do something just in case"*. The SS captures the dynamics of the problem and this is worth a balanced fleet in all scenarios in the second stage. It would lead the shipping company to charter on average 1.18 ships per scenario in $t = 1$, against the 1.66 per scenario of the DS. Furthermore, because of the limited number of charters available, the DS would lead to use voyage charters (the most expensive shipping alternative) in five of the fifteen scenarios whereas in the SS case, voyage charters would be used in only two scenarios. The average amount of cargo per scenario shipped by voyage charters would be 71055 RT43 for the DS against only 3904 RT43 for the SS.

However, even if the DS performs worse than the SS in terms of expectation, it still brings valuable pieces of information. We can notice that the two solutions look very similar to each other except for the sales. The DS is identical to the SS with regard to new buildings. It suggests exactly the same number and types of ships to build. Furthermore, it also chooses the same ship types to buy in the second-hand market, but not the same number of ships. These considerations are true independently of the scenario tree used to determine the SS, as they hold for all the SSs calculated in the tests.

From the above considerations we learn that a stochastic programming approach provided solutions more balanced against all the future scenarios and that this gives a much lower expected cost. The fleet created is well balanced and the amount of cargo transported by voyage charters in the second stage is limited. The DS is not able to capture the dynamics of the problem and thus it does not properly estimate the fleet size leading to imbalances in the second stage. However, the DS is able to capture the right mix of ships. Information coming from the DS might therefore still be useful, for example to simplify the stochastic program (e.g. by eliminating unnecessary variables that are determined from the MVP).

Table 5: First period solutions to the stochastic and deterministic problem for instance 6_5.

	Ship type	SS	DS
Initial fleet	LCTC2	9	9
	RORO2	8	8
	PCTC2	10	10
Second hand purchases	LCTC2	5	3
	RORO2	0	0
	PCTC2	0	0
Sales	LCTC2	0	0
	RORO2	0	3
	PCTC2	0	3
Scrapping	LCTC2	0	0
	RORO2	0	0
	PCTC2	0	0
Buildings	RORO1	4	4
	LCTC1	2	2
	PCTC1	0	0

6.4 Influence of charter limits and frequency requirements

Although Section 6.3 shows, for the case of WWL, the importance of modeling randomness in the MFRP, these results are not immediately generalizable. In this section we show how the VSS% changes with the number of available charters and with different service frequencies imposed on the trades. All the tests have been performed setting the highest loop cardinality to $C^{MAX} = 2$.

When testing the influence of the number of available charters we set up the set of controlled trades to $N_t^C = \emptyset, t = 1, \ldots, \bar{T}$. Figure 9 illustrates, for the different instances, how the VSS% diminishes when L^{CI} increases. The decrease is steeper for smaller instances (with correspondingly smaller tonnage requirements). However, shortages of charters (e.g. $L^{CI} = 1$) increase the risk more in the smaller instance than in the larger (i.e. the higher the VSS the higher the loss due to plans which ignore uncertanty). Big fleets give the possibility to compensate lack of charters by playing with the available ships more easily than small fleets. Charters represent the cheapest recourse decision in case of tonnage deficit. Lack of charters forces shipping companies to use more voyage charters. Therefore, as charters become more available, shipping companies can more easily recover from bad fleet planning decisions.

The above considerations suggest that big fleets are safer than small fleets in cases in which the market for time charters is very competitive and charters are scarcely available (as in the case of WWL). On the other hand, in more general cases, as the number of charters increases, small shipping companies benefit more than big shipping companies (i.e. the effect of incorrect renewals is less and less expensive). In general the importance of considering uncertainty in fleet renewal plans is lower the more active the charter market is. Example segments where charters are more easy to obtain are container shipping and general bulk shipping, where ships are more standardized and have a wider range of employment.

Finally, to test how the minimum service frequency imposed on the trades, H_{it}, influences the results, we applied the basic H_{it} given in Table 3 and gradually increased it while measuring the VSS%. The set of controlled trades is $N_t^C = N_t, t = 1, \ldots, \bar{T}$ for each instance and for these tests we set $L^{CI} = 3$ for all instances. Figure 10 shows how the VSS% decreases when the H_{it} is increased by 20% and then by 40%. As H_{it} increases the relevance of uncertainty decreases because the tonnage requirement is driven by the certain frequency imposed rather than by the uncertain demand. Liner shipping companies in which the

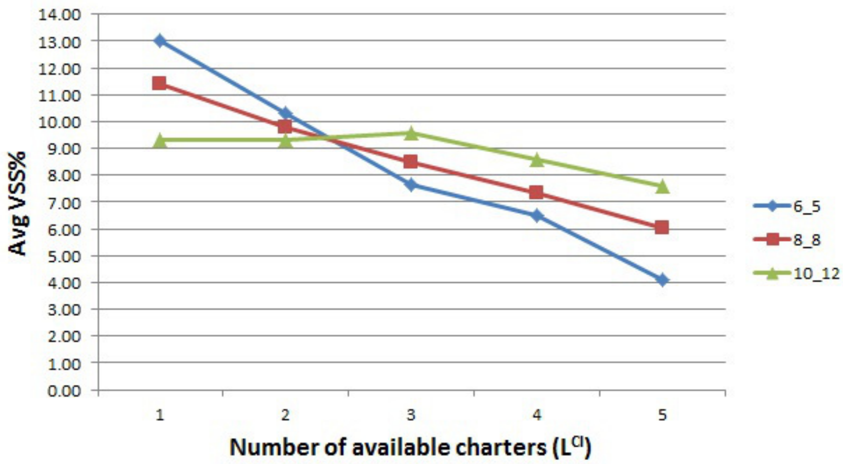

Figure 9: VSS% and number of charters available.

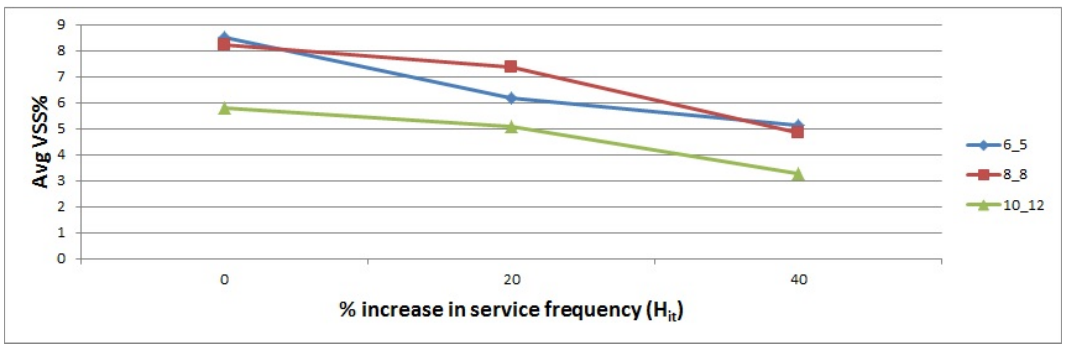

Figure 10: VSS% and frequency required.

fleet deployment is driven by frequency requirements are not unlikely. In such situations the relevance of uncertainty is gradually cut off as the frequencies imposed increase.

6.5 Influence of the deployment details

Finally, in this section we test how varying the level of detail in the deployment modeling influences the results. Increasing C^{MAX} corresponds to increasing the deployment level by including more and longer loops, which in turn implies that deployment decisions are modeled more accurately. As an example, if $C^{MAX} = 2$ the sets of loops $R_t, t = 1, \ldots, \bar{T}$ consists of loops including both one trade and two trades. If we increase it to $C^{MAX} = 3$ we also consider the loops made of three trades.

Table 6 reports the average values of the objective function (i.e. the total expected cost) given by the MFRP^{W} for different values of C^{MAX}. As expected, the objective value decreases when C^{MAX} increases due to the fact that more deployment alternatives become available. It can be noticed that using $C^{MAX} = 1$ gives very poor results due to the low level of details in the modeling of the deployment. Increasing to $C^{MAX} = 2$ gives significant improvements in the solutions by substantially reducing the ballast sailings. It should also be remarked that bigger values of C^{MAX} give higher solution times. It takes, for example, nearly seven hours on average to solve instance 10_12 with $C^{MAX} = 3$, while the corresponding time with $C^{MAX} = 2$ was on average only 40 minutes. These results justify our use of $C^{MAX} = 2$ throughout Section 6 as it gives a good compromise between computation time and solution quality. When using $C^{MAX} = 4$, in instances 6_5 and 8_8 we achieved only small improvements with respect to using $C^{MAX} = 3$. However, the cost reductions registered are not higher than the corresponding in sample variation, therefore the numbers reported might be the result of the numerical difference between different scenario trees. This applies also

Table 6: Expected cost and deployment information. Optimal objective values are expressed as a percentage of the optimal objective value for the case $C^{MAX} = 2$.

	Optimal objective value %		
C^{MAX}	6_5	8_8	10_12
1	167.23	163.24	168.52
2	100.00	100.00	100.00
3	99.95	99.75	98.49
4	99.93	99.70	98.64
5	99.94	99.73	-

to the cost increase reported for instance 10_12 with $C^{MAX} = 4$, and 6_5 and 8_8 with $C^{MAX} = 5$. In fact, the cost increase may not imply that the solutions they produce are worse. It is instead, possibly, simply noise from the scenario generation. In any case, the additional routes added when we increase the cardinality of the loops are very seldom used. Furthermore, using $C^{MAX} = 4$ and higher raises raises computational issues. Particularly, we did not always obtain a solution when solving instance 10_12 using $C^{MAX} = 4$, and we were not able to solve instance 10_12 with $C^{MAX} = 5$. Additional information on the computational problems are given in Section 6.6.

6.6 Numerical difficulties and possible solutions

We performed the tests presented throughout Section 6 by solving the deterministic equivalent of the stochastic programs. Although Xpress 7.2.1 solved almost all instances we presented quite efficiently, computational difficulties arise when the size of the instances increase. Table 7 reports the size of the instances for different values of C^{MAX}, in terms of number variables and constraints. It can be noticed that increasing C^{MAX} the number of continuous variables and constraints also increase, due to the addition of variables of type x_{vrt} and constraints of type (25)-(26). The number of integer variables increases instead with increases in the number of ship types and, in the instances we solved, is not extremely high. We observe that the problems are solved in a few branch-and-bound nodes. This is due to good bounds provided by solving the linear relaxation of the problem, and to the efficiency of the heuristic strategies employed by the solver in finding feasible integer solutions from the fractional solution to the linear relaxation. However, when the size of the problem increases, even solving the linear relaxation becomes increasingly time consuming. In addition, when solving instance 10_12 with $C^{MAX} = 4$ we did not always obtain a solution to the problem. The chances to obtain a solution increase with the quality of the initial solution when the solver starts the branch-and-bound method. If the optimality gap of the initial solution is too high, many times the branch-and-bound tree expands until the programs runs out of memory. Therefore, more efficient solution methods must intervene in order to face dimensionality problems. Below we sketch a possible strategy towards that.

Table 7: Size of the instances in terms of number of variables (Var) of which integer (Int) and number of constraints (Ctr). The size of the instance 10_12 with $C^{MAX} = 5$ is estimated.

C^{MAX}		6_5	8_8	10_12
2	Var(Int)	32 343(2 865)	91 238(3 963)	234 778(5 061)
	Ctr	37 716	110 225	288 559
3	Var(Int)	48 543(2 865)	215558(3 963)	855178(5 061)
	Ctr	57 966	265 625	1 064 059
4	Var(Int)	56 643(2 865)	370 958(3 963)	2 251 078(5 061)
	Ctr	68 091	459 875	2 808 934
5	Var(Int)	58 263(2 865)	495 278(3 963)	$\geq$ 3 500 000(5 061)
	Ctr	70 116	615 275	$\geq$4 300 000

We observe that, if the integer variables are fixed, the constraints modeling the fleet deployment (i.e. (8)-(9), (10)-(11) and (24)-(27) plus optional frequency and charter limit constraints), at each period and scenario, are independent from each other. This means that once the fleet renewal variables have been fixed (i.e. y^B_{vts}, y^{SH}_{kvts}, y^{SE}_{vts} and y^{SC}_{vts}), it becomes possible to make fleet deployment decisions for each time and scenario in separated problems. A possible way to exploit this feature is to obtain a decomposition of the problem into a (mixed-integer) master problem where fleet renewal decisions for all time periods and scenarios are made, and one subproblem for each combination time period-scenario in which deployment decisions are made. The deployment problems would be just (possibly small) linear programs. A solution to the MFRP would consist therefore of a fleet renewal solution (the solution to the master problem) and a set of fleet deployment solutions (the solutions to the subproblems). A possible algorithm could consist of heuristically generating fleet renewal decisions, and solving the corresponding fleet deployment problems in order to evaluate the total expected cost of the solution. The heuristic generation of fleet renewal decisions could start from the solution to the mean value problem which provides good solutions in terms of fleet mix (see Section 6.3). The heuristic could then focus on proposing solution to the master problem with better fleet size and (possibly) mix.

7 Conclusions

The renewal of a fleet of ships is an important task highly affected by uncertainty. We present a stochastic programming model for the problem to explicitly take uncertainty into account. Our main contribution is that of analyzing whether or not the inclusion of uncertainty in the model leads to better decisions. Furthermore, we investigate what circumstances strengthen or weaken the role of uncertainty. The model has been slightly adapted and applied to the case of Wallenius Wilhelmsen Logistics, a major liner shipping company.

Results show that the benefits of using stochastic programming are tangible compared to solving a deterministic model using expected values of the uncertain parameters. The solutions to the stochastic programming problem better capture the tonnage requirement needed to be prepared for the possible future market conditions described by the scenarios. Furthermore, we show that the importance of uncertainty in the problem diminishes in markets with easier access to charters and more standardized service frequencies. Moreover, large fleets are more resilient against bad fleet renewal plans due to the flexibility given by the higher number of ships.

The mean value problem still brings valuable pieces of information as it was always able to capture the right type of ships to invest in (i.e. fleet mix) although the fleet size was poorly estimated. This information might be useful to reduce the complexity of the stochastic program.

Acknowledgment

We want to thank Dr. Michal Kaut for providing his code for the scenario generation heuristic and precious support in the initial setup. We also thank the tree anonymous reviewers for their valuable comments. This work is partly financed by the MARFLIX (MARitime FLeet size and mIX) project launched by MARINTEK, NTNU and DNV, with financial support from the Research Council of Norway.

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