Formulations of a carsharing pricing and relocation problem

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Abstract. This article presents and compares two formulations for a pricing-based carsharing relocation problem. Given a target planning period, the problem consists of deciding simultaneously the price of carsharing rides between different zones of the city and the relocations of vehicles to perform to better serve demand. Customers response to pricing decisions are captured by utility functions. Results illustrate that one of the two formulations is superior in terms of ease of solution and scalability.

Keywords: Carsharing · Relocation · Pricing

1 Introduction

The increase in car ownership and usage, coupled with high dependency on private vehicles and low occupancy rates, have determined serious traffic congestion problems in many cities of the world [10] resulting pollution and poor urban air quality. Improvements of public transport [25] and road pricing measures [9,13] have, to a large extent, failed to provide sustainable solutions [10,2,21]. In this context, shared mobility, and particularly carsharing, has emerged as a viable alternative as it is linked to a decrease in congestion [12], pollution, land used [26] and transport costs [14,23].

Nevertheless, the attractiveness of carsharing systems, and their potential to replace car ownership, is heavily dependent on the level of service offered (e.g., the actual availability of vehicles when needed, and their distance from the user's location) and its cost [6]. Ensuring the necessary levels of service in an economically viable manner poses novel complex planning problems to carsharing operators (CSOs). Failure to deal with this complexity results in early failure such as those reported in [1,27,11]. The focal problem in this article is that of pricing carsharing rides and ensuring a spatial distribution of the fleet that complies with demand.

A central challenge faced in such systems is that one-way rentals, coupled with demand tides and oscillations [30,31], create frequent imbalances in the distribution of vehicles. This results in an accumulation of vehicles in low-demand zones, and vehicle shortage in high-demand zones [4,6] with levels of service dropping accordingly. Failure to ensure a spatial distribution of vehicles consistent with demand determines unreliable levels of service.

The research literature is slowly catching up and offering analytics methods for facing such challenges. Methods have been proposed for planning staff-based relocation activities [7,31,6,8,18,20,19,24,4,3], possibly combined with recharging [15] (see also the surveys [23,17]). These methods, typically based on optimization techniques, employ a mathematical problem (e.g., a MILP problem) to decide which vehicles should be relocated and where, and possibly which operators should perform the relocations. Methods have also been proposed for pricing carsharing services [16] and for inducing user-based relocations through pricing strategies [4,29,28].

This article addresses the problem of simultaneously setting pricing and deciding relocation activities. We propose two alternative formulations, the former derived from the demand-based discrete optimization framework of [5] also used in the context of carsharing by [16], the latter a pure IP reformulation of the former. We compare them in a computational study with the scope of identifying the one offering better performances in terms of solve times. The use of the two formulations is envisaged in the context of scenario analysis or simulation.

In Section 2 we provide a brief description of the problem followed by modeling assumptions. In Section 3 we present an extensive formulation in a discursive manner in order to clarify all details of the problem. In Section 4 we present a compact formulation. In Section 5 we describe a computational study and its results. Finally, we draw conclusions in Section 6.

2 Problem overview and modeling assumptions

Given a *target period* representing a portion of the day, e.g., a number of hours in the afternoon or morning, the distribution of vehicles at the time of planning, and the cumulative transport demand outlook for the target period, we address the problem of determining the prices to offer in the target period and the relocations to perform in preparation for the target period. Prices must be set taking into account customers preferences and the competition of alternative transport services (e.g., bus, metro, bicycle) and can vary with the origin and destination of the carsharing ride. The following assumptions are made.

- **A1** The price is made of a *drop-off fee*, which depends on the origin and destination of the rental, plus a per-minute fee which is identical to all zones. This is consistent with current pricing schemes in a number of carsharing services[1]. The drop-off fee may be negative to encourage desired movements of cars.
- **A2** The CSO can adjust the drop-off fee during the day, e.g., in response to demand waves.
- **A3** The CSO is able to inform customers about the current price from their location to every other zone, prior to rentals.

[1] See e.g., the pricing model recently adopted by Car2Go https://www.car2go.com/IT/en/milano/costs/.

A4 Alternative transport services (e.g., public transport and personal bicycles) have unlimited capacity.
A5 A customer chooses exactly one transport service among the available ones (i.e., the market is closed) and, particularly, the one that gives them the highest utility.
A6 An outlook of the cumulative demand of transport between different zones of the city is available (e.g., a forecast point or historical realization).
A7 Customers traveling with shared cars drive directly from their origin to their destination zone.
A8 Both the CSO and the customers are aware of all available transport services and of their characteristics (e.g., price, travel time and waiting time). Such characteristics are identical for all customers.

3 Extensive Formulation (F1)

Consider a urban area represented by a finite set $\mathcal{I}$ of zones and a CSO offering a finite set of shared vehicles $\mathcal{V}$. Before the beginning of the target period, the CSO is to decide the drop-off fee between each pair of zones and the relocations to perform to better serve demand in the target period.

At the time of planning, vehicles $v \in \mathcal{V}$ are geographically dispersed in the urban area as the result of previous rentals. Let parameter X_{vi} is equal to 1 if vehicle $v \in \mathcal{V}$ is initially in zone $i \in \mathcal{I}$, 0 otherwise, with $\sum_{i \in \mathcal{I}} X_{vi} = 1$ for all $v \in \mathcal{V}$. Let decision variable x_{vij} be equal to 1 if vehicle $v \in \mathcal{V}$ is relocated from zone $i \in \mathcal{I}$ to zone $j \in \mathcal{I}$, 0 otherwise. A vehicle can be relocated at most one time, that is

$$\sum_{j \in \mathcal{I}} x_{vij} \leq X_{vi} \qquad \forall v \in \mathcal{V}, i \in \mathcal{I} \tag{1a}$$

Let decision variable z_{vi} be equal to 1 if vehicle v is available for rental at zone i in the target period, 0 otherwise. It must hold that

$$z_{vi} = X_{vi} - \sum_{j \in \mathcal{I}} x_{vij} + \sum_{j \in \mathcal{I}} x_{vji} \qquad \forall v \in \mathcal{V}, i \in \mathcal{I} \tag{1b}$$

Let $\mathcal{L}$ be a finite set of drop-off fees the CSO is considering, and let decision variable λ_{ijl} be equal to 1 if fee l is applied when renting a car in zone i and leaving it in zone j, 0 otherwise. Only one drop-off fee can be selected between each pair of zones

$$\sum_{l \in \mathcal{L}} \lambda_{ijl} = 1 \qquad \forall i, j \in \mathcal{I} \tag{1c}$$

The city counts a set $\mathcal{A}$ of alternative transport services such as metro, bus, bicycle, and taxi. Let decision variable p_{vij} be the price of service $v \in \mathcal{V} \cup \mathcal{A}$ between zones i and j. The price of a carsharing ride between zones i and j is

$$p_{vij} = P_v^V T_{vij}^{CS} + \sum_{l \in \mathcal{L}} L_l \lambda_{ijl} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I} \tag{1d}$$

where parameter P_v^V is the per-minute fee of vehicle $v \in \mathcal{V}$, T_{vij}^{CS} the driving time between zones i and j and L_l the value of drop-off fee at level $l \in \mathcal{L}$. Instead, the price of alternative services is set as

$$p_{vij} = P_{vij} \qquad \forall v \in \mathcal{A}, i, j \in \mathcal{I} \tag{1e}$$

where parameter P_{vij} is the price of alternative service $v \in \mathcal{A}$ between i and $j \in \mathcal{I}$.

Let $\mathcal{K}$ be the set of customers, with $\mathcal{K}_i \subseteq \mathcal{K}$ being the set of customers traveling from zone $i \in \mathcal{I}$ and $\mathcal{K}_{ij} \subseteq \mathcal{K}_i$ the set of customers traveling from $i \in \mathcal{I}$ to $j \in \mathcal{I}$ in the target period. Each customer is uniquely characterized by their preferences. The preferences of customer k are described by a utility function $F_k(p_{vij}, \pi_{vij}^1, \ldots, \pi_{vij}^N)$ of the price p_{vij} and a number of characteristics $\pi_{vij}^1, \ldots, \pi_{vij}^N$ of transport service $v \in \mathcal{V} \cup \mathcal{A}$ between zones i and $j \in \mathcal{I}$ (e.g., travel and waiting time), and a random term $\tilde{\xi}_{kv}$ representing the portion of the preferences of customer k that the CSO is not able to describe by $F_k(\cdot)$. Any distribution for $\tilde{\xi}_{kv}$ is valid, leading in turn to different choice models such as the Logit model when $\tilde{\xi}_{kv}$ follows an extreme value distribution (see [5]). Let u_{ijkv} be a decision variable representing utility obtained by customer $k \in \mathcal{K}$ when moving from i to $j \in \mathcal{I}$ using service $v \in \mathcal{V} \cup \mathcal{A}$. The utility is determined by

$$u_{ijkv} = F_k(p_{vij}, \pi_{vij}^1, \ldots, \pi_{vij}^N) + \xi_{kv} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v \in \mathcal{V} \cup \mathcal{A} \tag{1f}$$

where ξ_{kv} is a realization of $\tilde{\xi}_{kv}$. Constraints (1f) are linear if $F_k(\cdot)$ is linear in p_{vij}.

Let binary variable y_{ikv} be equal to 1 if service $v \in \mathcal{V} \cup \mathcal{A}$ is offered to customer $k \in \mathcal{K}_i$, 0 otherwise. Alternative services $v \in \mathcal{A}$ are always offered to customers whenever they are available at all, that is

$$y_{ikv} = Y_{vi} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{A} \tag{1g}$$

where parameter Y_{vi} is equal to 1 if alternative service v is available in zone i, 0 otherwise. Conversely, a shared car $v \in \mathcal{V}$ can be offered to customers in zone i whenever it is physically available at i, that is

$$y_{ikv} \leq z_{iv} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{V} \tag{1h}$$

Let decision variable w_{ijkv} be equal to 1 if customer $k \in \mathcal{K}_{ij}$ chooses service $v \in \mathcal{V} \cup \mathcal{A}$, 0 otherwise. A customer will choose exactly one service

$$\sum_{v \in \mathcal{V} \cup \mathcal{A}} w_{ijkv} = 1 \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij} \tag{1i}$$

And a service can be chosen only if it is offered to the customer

$$w_{ijkv} \leq y_{ikv} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v \in \mathcal{V} \cup \mathcal{A} \tag{1j}$$

Among the available services, the customer will chose the one yielding the highest utility. Therefore, for a given zone $i \in \mathcal{I}$, let decision variable ν_{ivwk} be equal to 1 if both services v and w in $\mathcal{V} \cup \mathcal{A}$ are available to customer $k \in \mathcal{K}_i$, 0 otherwise, and decision variable μ_{ijvwk} be equal to one if service $v \in \mathcal{V} \cup \mathcal{A}$ yields a greater utility than service $w \in \mathcal{V} \cup \mathcal{A}$ to customer $k \in \mathcal{K}_{ij}$ moving from i to j, 0 otherwise. The following constraints state that ν_{ivwk} is equal to one when both services v and w are available

$$y_{ikv} + y_{ikw} \leq 1 + \nu_{ivwk} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v, w \in \mathcal{V} \cup \mathcal{A}, \tag{1k}$$

$$\nu_{ivwk} \leq y_{ikv} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v, w \in \mathcal{V} \cup \mathcal{A}, \tag{1l}$$

$$\nu_{ivwk} \leq y_{ikw} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v, w \in \mathcal{V} \cup \mathcal{A}. \tag{1m}$$

A service is chosen only if it yields the highest utility

$$w_{ijkv} \leq \mu_{ijvwk} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A} \tag{1n}$$

that is, as soon as μ_{ijvwk} is set to 0 for some index w, w_{ijkv} is forced to 0 and service v is not chosen by customer k on i-j. The following constraints ensure that decision variable μ_{ijvwk} takes the correct value according to the utility

$$M_{ijk}\nu_{ivwk} - 2M_{ijk} \leq u_{ijkv} - u_{ijkw} - M_{ijk}\mu_{ijvwk} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A} \tag{1o}$$

and

$$u_{ijkv} - u_{ijkw} - M_{ijk}\mu_{ijvwk} \leq (1 - \nu_{ivwk})M_{ijk} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A} \tag{1p}$$

where constant M_{ijk} represents the greatest difference in utility between two services between i and $j \in \mathcal{I}$ for customer $k \in \mathcal{K}_{ij}$, that is $M_{ijk} \geq |u_{ijkv} - u_{ijkw}|, \forall v, w \in \mathcal{V} \cup \mathcal{A}$. Constraints (1o)-(1p) work as follows. When both services v and w are available ($\nu_{ivwk} = 1$) and $u_{ijkv} > u_{ijkw}$, (1p) forces μ_{ijvwk} to take value 1, while (1o) reduces to $0 \leq u_{ijkv} - u_{ijkw}$. When both service v and w are available and $u_{ijkv} < u_{ijkw}$, (1o) forces μ_{ijvwk} to take value 0, while (1p) reduces to $0 \geq u_{ijkv} - u_{ijkw}$. When one of the two services is not available ($\nu_{ivwk} = 0$), constraints (1o)-(1p) are satisfied irrespective of the value of μ_{ijvwk}. In case of ties ($u_{ijkv} = u_{ijkw}$) we impose

$$\mu_{ijvwk} + \mu_{ijvwk} \leq 1 \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A} \tag{1q}$$

A service can be preferred only if offered

$$\mu_{ijvwk} \leq y_{ikv} \qquad \forall i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, v, w \in \mathcal{V} \cup \mathcal{A} \tag{1r}$$

Let decision variable α_{ijkvl} be equal to 1 if fare l is applied between i and j and customer k chooses shared car $v \in \mathcal{V}$, 0 otherwise. The following constraints ensure the relationship between λ_{ijl} and w_{ijkv} and α_{ijkvl}

$$\lambda_{ijl} + w_{ijkv} \leq 1 + \alpha_{ijkvl} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, l \in \mathcal{L} \tag{1s}$$

$$\alpha_{ijkvl} \leq \lambda_{ijl} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, l \in \mathcal{L} \tag{1t}$$

$$\alpha_{ijkvl} \leq w_{ijkv} \qquad \forall v \in \mathcal{V}, i, j \in \mathcal{I}, k \in \mathcal{K}_{ij}, l \in \mathcal{L} \tag{1u}$$

Each car $v \in \mathcal{V}$ can accommodate only one customer. If more than one customers wish to use car v, the car is taken by the first customer arriving at the car. Assuming that customers are indexed according to their arrival time at the car, i.e., customer k arrives before $k+1$, we impose that a vehicle is offered to a customer only if it is offered also to the customer arriving before them (who perhaps did not take it), that is:

$$y_{ikv} \leq y_{i(k-1)v} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{V} \tag{1v}$$

A vehicle becomes unavailable for a customer if any customer has arrived before them and rented the car, that is:

$$z_{iv} - y_{ikv} = \sum_{j \in \mathcal{I}} \sum_{q \in \mathcal{K}_{ij} : q < k} w_{ijqv} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K}_i, v \in \mathcal{V} \tag{1w}$$

that is, if car v is in zone i ($z_{iv} = 1$), but it is not offered to customer k ($y_{ikv} = 0$) we obtain

$$1 = \sum_{j \in \mathcal{I}} \sum_{q \in \mathcal{K}_{ij} : q < k} w_{ijqv}$$

meaning that one customer has arrived before k and rented the car. On the other hand, if the car is offered to customer k, ($y_{ikv} = 1$), then it must be in zone i ($z_{iv} = 1$ – see (1h)), and we obtain

$$0 = \sum_{j \in \mathcal{I}} \sum_{q \in \mathcal{K}_{ij} : q < k} w_{ijqv}$$

meaning that no customer arriving before k has taken the car. The same equality holds if the vehicle is not available at all ($z_{iv} = 0$ and $y_{ijkv} = 0$).

The CSO maximizes their profit by means of the following objective function

$$\max \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{I} \times \mathcal{I}} \left(P_v^V T_{vij}^{CS} - C_{vij}^U \right) \sum_{k \in \mathcal{K}_{ij}} w_{ijkv} \tag{1x}$$

$$+ \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{I} \times \mathcal{I}} \sum_{k \in \mathcal{K}_{ij}} \sum_{l \in \mathcal{L}} L_{ijl} \alpha_{ijkvl} \tag{1y}$$

$$- \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{I} \times \mathcal{I}} C_{vij}^R x_{vij} \tag{1z}$$

where C_{vij}^U is the cost born by the CSO when vehicle v is rented between i and j and (1x) represents the net revenue generated by the per-minute fee, (1y) represents the revenue generated by the drop-off fee, C_{vij}^R is the cost of relocating vehicle v from i to j and (1z) represents the total relocation cost.

Therefore, formulation F1 consists of objective function (1x)–(1z) subject to (1a)–(1w).

4 Compact formulation (F2)

Given a realization ξ_{kv} of $\tilde{\xi}_{kv}$, e.g., in a scenario analysis, a compact reformulation of the problem can be derived by preprocessing customers preferences. This formulation has a double advantage: its size is in general smaller than F1, and it does not require the utility function to be linear in the price.

We introduce the concept of a request. A request represents a customer who wishes to use carsharing for moving from its origin to its destination. Let $\mathcal{R}$ be the set requests. The set $\mathcal{R}$ contains a request for each customer $k \in \mathcal{K}$ for which there exists at least one drop-off level $l \in \mathcal{L}$ such that the customer would prefer carsharing to alternative transport services that is, for which $u_{ijkv} > u_{ijkw}$ with $v \in \mathcal{V}$ and $w \in \mathcal{A}$ (note that all shared cars yield the same utility). Let $i(r)$, $j(r)$ and $k(r)$ be the origin, destination and customer associated with request r, respectively, and $l(r)$ the highest drop-off fee at which customer $k(r)$ would prefer carsharing to other services. Note, that customer $k(r)$ would still prefer carsharing at any drop-off fee lower than $l(r)$ (under the reasonable assumption that the customer is sensitive to price). For each $r \in \mathcal{R}$ let $R_{vrl} = P_v^V T^{CS}_{v,i(r),j(r)} - C^U_{v,i(r),j(r)} + L_l$, for $l \leq l(r)$, be the profit generated if request r is satisfied by vehicle v with drop-off fee l. Let $C^R_{vi} = C^R_{vji}$ if v is initially in $j \neq i$, 0 otherwise, be the cost of making vehicle v available at i. Let $\mathcal{R}_r = \{\rho \in \mathcal{R} : i(\rho) = i(r), k(\rho) < k(r)\}$ be the set of requests which have a precedence over r. Let $\mathcal{R}_{ij} = \{r \in \mathcal{R} : i(r) = i, j(r) = j\}$. Let $\mathcal{L}_r = \{l \in \mathcal{L} : l \leq l(r)\}$. Finally, let decision variable y_{vrl} be equal to 1 if request r is satisfied by vehicle v at level l, 0 otherwise. Let z_{vi} if vehicle v is made available at zone i, 0 otherwise. Finally, let λ_{ijl} be equal to 1 if drop-off level l is applied between i and j, 0 otherwise. Formulation F2 is hence

$$\max \sum_{r\in\mathcal{R}} \sum_{v\in\mathcal{V}} \sum_{l\in\mathcal{L}_r} R_{vrl} y_{vrl} - \sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{I}} C^R_{vi} z_{vi} \tag{2a}$$

$$\sum_{v\in\mathcal{V}} \sum_{l\in\mathcal{L}_r} y_{vrl} \leq 1 \qquad r \in \mathcal{R} \tag{2b}$$

$$\sum_{r\in\mathcal{R}} \sum_{l\in\mathcal{L}_r} y_{vrl} \leq 1 \qquad v \in \mathcal{V} \tag{2c}$$

$$\sum_{i\in\mathcal{I}} z_{vi} = 1 \qquad v \in \mathcal{V} \tag{2d}$$

$$\sum_{l\in\mathcal{L}_{r_1}} y_{v,r_1,l} - z_{v,i(r_1)} + \sum_{r_2\in\mathcal{R}_{r_1}} \sum_{l\in\mathcal{L}_{r_2}} y_{v,r_2,l} \leq 0 \qquad r_1 \in \mathcal{R}, v \in \mathcal{V} \tag{2e}$$

$$\begin{aligned} y_{v,r_1,l_1} \geq{}& \lambda_{i(r_1),j(r_j),l_1} + z_{v,i(r_1)} \\ &- \sum_{r_2\in\mathcal{R}_{r_1}} \sum_{l_2\in\mathcal{L}_{r_2}} y_{v,r_2,l_2} - \sum_{v_1\in\mathcal{V}:v_1\neq v} y_{v_1,r_1,l_1} - 1 \qquad r_1 \in \mathcal{R}, v \in \mathcal{V}, l_1 \in \mathcal{L}_{r_1} \end{aligned} \tag{2f}$$

$$\sum_{l\in\mathcal{L}} \lambda_{ijl} = 1 \qquad i \in \mathcal{I}, j \in \mathcal{J} \tag{2g}$$

$$\sum_{v \in \mathcal{V}} y_{vrl} \leq \lambda_{i(r),j(r),l} \qquad r \in \mathcal{R}, l \in \mathcal{L}_r \quad \text{(2h)}$$

$$y_{vrl} \in \{0,1\} \qquad r \in \mathcal{R}, v \in \mathcal{V}, l \in \mathcal{L}_r \quad \text{(2i)}$$

$$z_{vi} \in \{0,1\} \qquad i \in \mathcal{I}, v \in \mathcal{V} \quad \text{(2j)}$$

$$\lambda_{ijl} \in \{0,1\} \qquad i \in \mathcal{I}, j \in \mathcal{I}, l \in \mathcal{L}. \quad \text{(2k)}$$

The objective function (2a) maximizes the profit obtained by the satisfaction of customer requests minus the cost of relocating vehicles. Constraints (2b) ensure that each request is satisfied at most once and constraints (2c) that each vehicle satisfies at most one request. Constraints (2d) ensure that a vehicle is available in exactly one zone. Constraints (2e) state that a request can be satisfied by vehicle v only if the vehicle is in zone $i(r_1)$ and the vehicle has not been assigned to a customer with a lower index (that is, arriving at the vehicle before $k(r_1)$). Constraints (2f) state that a request r_1 at a certain level l_1 must be satisfied by a vehicle v if the fare level l_1 has been selected and the vehicle is available at $i(r_1)$, unless the car has been used to satisfy the request of a customer with a higher priority, or r_1 has been satisfied by another vehicle. Constraints (2g) state that for each i and j only one drop-off fee can be applied. Finally, constraints (2h) states that a request can be satisfied at level l only if fare l is applied to all customers traveling between i and j.

5 Computational Study

Formulations F1 and F2 include a number of arbitrary elements subject to uncertainty, such as customers unknown preferences ξ_{kv}, their location and destination. As such, the envisaged usage of F1 and F2 is within a simulator, scenario analysis or as a component of a larger stochastic program, where different scenarios of the uncertain elements are assessed. Therefore, the scope of the computational study is to compare the two formulations on the case study illustrated in Section 5.1 in terms of solve time, percentage of problems solved, tightness of their LP relaxation and size. Results are presented in Section 5.2.

5.1 Case Study

To compare the two formulation we use a case study that replicates the carsharing system in the city of Milan. We assume the decision maker is a CSO with a homogeneous fleet $\mathcal{V} = \{1, \ldots, V\}$, servicing customers $\mathcal{K} = \{1, \ldots, K\}$. The alternative transport services are *public transport* (PT – bus and metro) and *bicycles* (B). Therefore we set $\mathcal{A} = \{PT, B\}$. A discretization of the business area of the city of Milan into ten zones is provided by [16], thus $\mathcal{I} = \{1, \ldots, 10\}$.

To each vehicle v we randomly assign an initial zone i (parameter X_{vi}). Similarly, we randomly partition customers into sets $\mathcal{K}_i$ and then further into sets $\mathcal{K}_{ij}$. Each customer k is characterized by a unique utility function. We adopt a variant of the utility function provided by [16] which further elaborates the utility function provided by [22]. The function is linear in the price rendering

F1 a mixed-integer linear program. For each customer $k \in \mathcal{K}$ traveling between i and j with transport service v, the utility can be stated as (3).

$$F_k(p_{vij}, T^{CS}_{vij}, T^{PT}_{vij}, T^{B}_{vij}, T^{Walk}_{vkij}, T^{Wait}_{vij}) = \beta^P_k p_{vij} + \beta^{CS}_k T^{CS}_{vij} + \beta^{PT}_k T^{PT}_{vij} + \tau(T^B_{vij})\beta^B_k T^B_{vij} + \tau(T^{Walk}_{vij})\beta^{Walk}_k T^{Walk}_{vij} + \beta^{Wait}_k T^{Wait}_{vij} \quad (3)$$

where

- T^{CS}_{vij} is the spent riding a shared car between i and j when using service v. This quantity is strictly positive only when v is a carsharing service, otherwise it is 0.
- T^{PT}_{vij} is the time spent in public transport between i and j when using service v. This quantity is strictly positive only when v is PT, otherwise it is 0.
- T^{B}_{vij} is the time spent riding a bicycle between i and j when using service v. This quantity is strictly positive only when v is B, otherwise it is 0.
- T^{Walk}_{vkij} is the walking time necessary for customer k to move with transport service v between i and j. This includes the walking time to the nearest service (e.g., shared car or bus stop), between connecting means (e.g., when switching between bus and metro to reach the final destination), and from to the final destination.
- T^{Wait}_{vij} is the total waiting time when using service v between i and j, and includes the waiting time for the service (e.g., bus or metro) as well as for connection.

The function $\tau : \mathbb{R} \to \mathbb{R}$, defined as $\tau(t) = \lceil \frac{t}{5} \rceil$, allows us to model the utility of cycling and walking as a piece-wise linear function: the utility of walking and cycling decreases faster as the walking and cycling time increases. Coefficients β^P_k, β^{CS}_k, β^{PT}_k, β^B_k, β^{Walk}_k and β^{Wait}_k represent the sensitivity of customer k to price, carsharing ride time, time spent in public transport service, cycling time, walking and waiting time, respectively.

For each (i, j) pair [16] provide a specification in minutes of the above mentioned T-parameters calculated on the actual services in Milan. In addition [16] provide base values for the β coefficients following the procedure illustrated by [22]. Particularly, they set $\beta^{CS} = -1$, $\beta^{PT} = -2$, $\beta^B = -2.5$, $\beta^{Walk} = -3$ and $\beta^{Wait} = -6$ and $\beta^P = -188.33$ if a customer belongs to the *lower-middle class* or $\beta^P = -70.63$ if a customer belongs to the *upper-middle class*. In order to create K unique customers, each customer will be characterized by a perturbation of the β coefficients provided by [16]. Particularly, β^P_k will be uniformly drawn in $[-188.33, -70.63]$ in order to obtain customers between the upper- and lower-middle class and the remaining β coefficients will be uniformly drawn in $[0.8\beta, 1.2\beta]$, where β is the value provided by [16]. As an example, for each k we will draw β^{PT}_k in $[-1.6, -2.4]$.

The price of a bicycle ride is set to $P_{Bij} = 0$ for all (i, j) pairs. Based on current prices in Milan we set $P_{PT,ij} = 2$ (in Euro) for all (i, j) and $P^V_v = 0.265$ Euros corresponding to the average carsharing per-minute fee in Milan. The drop-off fees considered are -2,-1,0,1, and 2 Euros in order to include the possibility that the company provides a bonus for the desired movements of cars. The relocation cost C^R_{vij} represents the cost of the fuel necessary for a ride between i and j (we assume Fiat 500 cars with classical combustion engine and consumption of $0.043l/h$) and the per-minute salary of the driver multiplied by the driving time. The fuel cost is calculated assuming an average speed of 50km/h and a fuel price of 1.60 Euro/l. The per-minute salary of the driver is calculated as the average of the last four retribution levels in the Italian national collective contract for logistics services valid at October 1st 2019[2] and amounts to approximately 0.11 Euros. The cost C^U_{vij} is set equal to the fuel necessary for a ride between i and j.

Finally, realizations of $\tilde{\xi}_{kv}$ are independently drawn from a Gumbel (Extreme Value type I) distribution with mean 0 and standard deviation σ calculated as the empirical standard deviation of $U_{ijkv} = F_k(p_{vij}, T^{CS}_{vij}, T^{PT}_{vij}, T^{B}_{vij}, T^{Walk}_{vij}, T^{Wait}_{vij})$ for all $i, j \in \mathcal{I}, v \in \mathcal{V} \cup \mathcal{A}, k \in \mathcal{K}_{ij}$.

Particularly, we solve a set of small instances with $V \in \{20, 35, 50\}$ and $K \in \{50, 75, 100\}$ and a set of medium instances with $V \in \{50, 75, 100\}$ and $K \in \{200, 300\}$. For each combination of V and K we randomly generate five instances (each of the five instances will be different in terms of position and characteristics of the customers and distribution of vehicles).

5.2 Results

Problems are solved with Cplex 12.10 on a machine equipped with CPU 2 x 2.4GHz AMD Opteron 2431 6 core and 24Gb RAM. A time limit of 360 seconds is set on all runs.

Table 1 reports the average solve time and percentage of problems solved for the small and medium instances. Already on the small instances it can be observed that F2 is superior to F1. F2 solves all problems to optimality (i.e., within the default Cplex 0.01% tolerance) in at most 2.635 seconds on average, while F1 solves only all the smallest instances and, also in that case, it spends a significantly higher amount of time compared to F2. On the medium instances F1 does not solve any of the problems, and in a number of cases the solution process fails due to an excessive use of memory resources. On the same instances, F2 solves all problems to optimality with an average solution time much smaller than the allocated time limit.

[2] https://www.lavoro-economia.it/ccnl/ccnl.aspx?c=328

Table 1: Average solve time [sec] and percentage of problems solved for the small instances. The symbol "–" indicates that the solution process failed for excessive consumption of memory resources.

		Time [sec]		Solved [%]	
V	K	F1	F2	F1	F2
20	50	67.485	0.465	100	100
20	75	187.209	0.506	80	100
20	100	309.379	0.658	40	100
35	50	342.699	0.784	20	100
35	75	360.953	1.230	0	100
35	100	362.098	1.771	0	100
50	50	361.400	1.432	0	100
50	75	361.060	2.048	0	100
50	100	363.038	2.635	0	100
50	200	–	6.763	–	100
50	300	–	13.680	–	100
75	200	368.963	10.964	0	100
75	300	397.119	18.955	0	100
100	200	384.174	19.546	0	100
100	300	376.794	34.848	0	100

The better performance of F2 is motivated by the tightness of its LP relaxation and its compact size. Regarding the quality of the LP relaxation, Table 2 reports the optimal objective value of the five randomly generated instances with $V = 20$ and $K = 50$ (the only instance size for which F1 solved all instances to optimality – see Table 1) together with the optimal objective value of the LP relaxations of F1 and F2. It can be noticed that F2 has a very strong LP relaxation as its optimal objective value corresponds to that of the IP formulation on all 5 instances in Table 2. At the same time, the LP relaxation of F1 provides a bound which is approximately from two to four times the optimal objective value. For the remaining instances not shown in Table 2, the LP gap for formulation F2 is zero on all the small instances, while on the medium instances it is zero for 24 of the 30 instances. For the remaining 6 instances the average LP gap is 0.018%, the maximum is 0.224% and the standard deviation is 0.052%. However, despite the relatively small LP gap, the solution to the LP relaxation is highly fractional, with limited chances obtaining the optimal solution by means of a simple rounding procedure.

The size of the two formulations also explains the different performances. The average size of the instances is reported in Table 3. The table illustrates how the size of F1 is orders of magnitude larger than the size of F2.

Table 2: Optimal objective value compared to the optimal objective value of the LP relaxations for the instances with $V = 20$ and $K = 50$.

Instance	V	K	Optimal objective value	LP objective value F1	LP objective value F2
1	20	50	53.58	124.22	53.58
2	20	50	40.64	128.12	40.64
3	20	50	25.94	117.19	25.94
4	20	50	25.94	119.35	25.94
5	20	50	38.44	124.75	38.44

Table 3: Average size of the instances in 10^4 variables/constraints.

V	K	# Variables F1	# Variables F2	# Binary F1	# Binary F2	# Constraints F1	# Constraints F2
20	50	5.24	0.17	5.14	0.17	19.48	0.16
20	75	7.87	0.21	7.72	0.21	29.67	0.22
20	100	10.41	0.25	10.21	0.25	39.63	0.27
35	50	14.10	0.26	13.94	0.26	53.74	0.26
35	75	21.22	0.34	20.97	0.34	81.61	0.37
35	100	28.13	0.41	27.79	0.41	108.72	0.46
50	50	27.25	0.35	27.01	0.35	104.92	0.36
50	75	41.04	0.46	40.68	0.46	159.12	0.51
50	100	54.41	0.56	53.94	0.56	211.70	0.65
50	200	109.05	1.01	108.10	1.01	432.08	1.26
50	300	164.23	1.46	162.79	1.46	660.95	1.87
75	200	235.16	1.49	233.75	1.49	931.39	1.88
75	300	354.17	2.16	352.05	2.16	1418.59	2.79
100	200	409.06	1.98	407.20	1.98	1619.71	2.49
100	300	616.12	2.87	613.31	2.87	2461.23	3.70

6 Conclusions

An extensive and a compact formulation for a carsharing pricing and relocation problem have been proposed. The formulations allow a carsharing operator set the price of carsharing rides between different zones of the city and at the same time the relocations to perform to better serve demand. Customers choices are directly included in the models by means of utility functions. The computational study shows that the compact formulation outperforms the extensive formulation in terms of solution time, number of instances solved and quality of the linear programming bound. Particularly, the relatively small solution time makes the compact formulation amenable to use in the context of a scenario analysis or simulation.

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