Maximizing the rate of return on the capital employed in shipping capacity renewal

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Abstract

Decisions regarding investments in capacity expansion/renewal require taking into account both the operating fitness and the financial performance of the investment. While several operating requirements have been considered in the operations research literature, the corresponding financial aspects have not received as much attention. We introduce a model for the renewal of shipping capacity which maximizes the Average Internal Rate of Return (AIRR). Maximizing the AIRR sets stricter return requirements on money expenditures than classic profit maximization models and may describe more closely shipping investors' preferences. The resulting nonlinear model is linearized to ease computation. Based on data from a shipping company we compare a profit maximization model with

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an AIRR maximization model. Results show that while maximizing profits results in aggressive expansions of the fleet, maximizing the return provides more balanced renewal strategies which may be preferable to most shipping investors.

1   Introduction

Among the most crucial decisions for a shipping company, the composition of the fleet of ships determines, to a great extent, the competitiveness of the company. Finding the best adaption of the fleet to volatile market conditions is the main scope of the Maritime Fleet Renewal Problem (MFRP), which consists of deciding how many and which types of ships to add to the fleet and which available ships to dispose of (see, e.g., Alvarez et al. (2011), Pantuso et al. (2014), and Pantuso et al. (2015a)).

The MFRP can be considered a special version of the Capacity Expansion Problem (CEP) or of the Machine Replacement Problem (MRP). CEPs seek the best addition to available capacity in order to meet increasing demand, while MRPs seek the best substitution of available machines, induced by factors such as obsolescence (Nair and Hopp, 1992), deterioration, and ageing. In CEPs and MRPs the terms "capacity" and "machine" generically refer to equipment of various types, such as, cables, pumps, computers, and vehicles (Rajagopalan, 1998), with differences in, for example, economic life, cost magnitude, and relevance for the core business.

CEPs and MRPs have received considerable attention by the operations research community, producing a plethora of models at increasing level of realism, and adapting to various operating configurations. For example, Fong and Srinivasan (1986) consider multi-period capacity expansion and location, Li and Tirupati (1994) focus on the trade-off between specialized and flexible capacity in multi-product production systems, Cormier and Gunn (1999) consider warehouse capacity expansion under inventory constraints, Kimms (1998) combines capacity expansion with production planning and lot sizing, van Ackere et al. (2013) study the short-term problem of adjusting the capacity in reaction to the behavior of customers waiting in queues, while Ahmed et al. (2003) and Bean et al. (1992) study CEPs under uncertainty. The main issues faced in CEPs are related to expansion size, time, and location (Luss, 1982) and the option of replacing machines is typically ignored (Rajagopalan, 1998).

2

As far as MRPs are concerned, Sethi and Chand (1979) consider the replacement of single machines with only one replacement alternative, while Chand and Sethi (1982) allow the possibility of replacing available machines by any from a set of available alternatives. Goldstein et al. (1988), Nair and Hopp (1992), Hopp and Nair (1994), and Adkins and Paxson (2013) consider replacement decisions triggered by technological breakthroughs. Typically, MRPs do not consider the possibility of changes in the demand for equipment. Capacity expansions and replacements are however naturally tied decisions (see, e.g., Rajagopalan and Soteriou (1994), Rajagopalan (1998), Rajagopalan et al. (1998), Chand et al. (2000)).

The problem of expanding/replacing transportation capacity takes on specific features due to the interplay between the investment in vehicles and their routing. Classical models focus mainly on the initial configuration of a fleet of vehicles (see, e.g., Fagerholt et al. (2010) and the surveys in Hoff et al. (2010) and Pantuso et al. (2014)), rather than its evolution. However, the problem of renewing fleets of vehicles has recently received attention especially in the maritime literature, due to the volatile nature of the shipping business, and the consequent need to adjust shipping capacities in response to changes in the market. As an example, Alvarez et al. (2011) and Pantuso et al. (2015a) consider multi-period renewal of a fleet of ships in order to cope with uncertain market developments. Examples can also be found for rail-road capacity expansions (see, e.g., Liu et al. (2008)).

The studies mentioned above cover a wide variety of operating features and equipment types. However, relatively little attention has been paid by the operations research community to the financial aspects related to investments in capacity besides their technical fitness. Most of the models available seek minimum cost or maximum profit capacity expansion/replacement decisions with the Net Present Value (NPV) being the only metric used. However, financial and economic data related to an investment can be aggregated in a number of alternative ways, giving rise to different metrics often used in place of, or in conjunction with, the NPV for evaluating the profitability of capital asset investments (see, e.g., Schall et al. (1978) and Magni (2015)). This is especially true for equipment with long economic life and a relevant capital magnitude, such as vehicles, buildings, and pipelines. As an example, Menezes et al. (2015), pointing out that a mere attention to profit in facility location can lead to too high investments, include Return on Investment thresholds requirements in the corresponding models, and show that this leads to a higher utilization of the available facilities. Particularly,

3

for the case of maritime shipping, Stopford (2009) shows that investments can be evaluated by the ratio between the economic value added by the transportation services over the net asset value of the fleet.

In this paper, we consider the maximization of the *Average Internal Rate of Return* (AIRR) in the renewal of maritime shipping capacity. The AIRR (Magni, 2010) measures the return of multi-period investment projects which generalizes and solves a number of flaws of the well known concept of *Internal Rate of Return* (IRR) as explained by Magni (2013). It can be expressed as the ratio between the actualized returns generated by a stream of capital invest-ments over the actualized sum of the investments. This metric is in line with the indicator used in Stopford (2009). The focus is on the MFRP as it well represents strategic CEPs and MRPs due to the long economic life of ships, their cost magnitude, and the high level of uncertainty. As an example, the second-hand price of a five year old 300 000 deadweight tons (dwt - a standard measurement unit for the ship carrying capacity) oil tanker, increased from 124 to 145 million dollars in 2008, and fell down again to 84 million dollars in 2009 as reported by the United Nations Conference on Trade and Development (UNCTAD, 2012). The contribution of this paper is therefore twofold: 1) we introduce a model for maximizing the AIRR for capacity renewal in shipping, and 2) we compare the results of the new model against that of a more classic model maximizing profits NPV in order to offer managerial insights by highlighting the economical and structural differences in the solutions obtained. In addition, we show how the resulting nonlinear AIRR model can be reformulated in an equivalent linear model in order to ease computation. In order to account for market infor-mation being revealed at different points in time, both the AIRR and the profit maximization problems are formulated as multistage stochastic programs.

The remainder of this paper is organized as follows. In Section 2 we provide a thorough description of the MFRP. In Section 3 we introduce a mathematical model for the MFRP which maximizes the AIRR, as well as an alternative model which maximizes profits. In Section 4 we analyze the results and the solutions obtained by the two alternative models based on the case of a major liner shipping company. Finally, conclusions are drawn in Section 5.

4

2 The Renewal of Maritime Shipping Capacity

The MFRP is a special version of MRPs and CEPs due to routing constraints. The objective is to seek an investment mix which is sound in some economical sense (typically cost efficient) and respects operating constraints. In what follows, we sketch the main features of the problem, while a detailed description can be found in Pantuso et al. (2015a).

The MFRP consists of deciding, for each time period, how many ships of each type to add to or remove from the available fleet. Ships can be bought in the second-hand market, or built. In the former case, the company must choose from the ships available in the market but the ship is available in short time (typically weeks to months). In the latter case the ship can be built according to the company's specifics but the building process takes longer time (typically years). Ship prices depend, to a great extent, on the type of ship, its age, and on the market status. Ships can be disposed of by selling them in the second-hand market or scrapping (demolishing) them. In both cases the ship can be removed from the fleet in weeks to months. Scrapping rates depend to a great extent on the weight of the steel the ship is made of, and are therefore sensitive to changes in steel prices.

A necessary distinction must be made. In the shipping business there exist two broad types of players interested in investing in ships, which we will refer to as *speculators* and *ship operators*. Speculators see ships as an asset to trade. Their main scope is buying ships in order to sell them at a higher price when the market allows so. They do not necessarily have competencies in shipping operations, but see ships as a marketable asset. Ship operators, on the contrary, buy ships to operate them. Their business model consists of using ships to provide transportation services. Finer classifications, though possible, are beyond the scope of this paper. In what follows we refer to the ship operator type of player.

When deciding how to modify the available fleet, investors must take into account how the fleet is operated. This includes both the possibility of temporary adjustments to the fleet and the utilization (i.e., the sailing activities) of the available fleet. Temporary adjustments to the fleet are mainly done by means of *time charters*, which consist of hiring a ship and its crew for a period time (weeks to years). The charterer pays a (per day) fee as well as all sailing-related expenses, such as fuel and port fees. The owner of the ship bears the rest of the costs, such as capital cost, crew, and insurance. Any shipping company can, in general,

act both as a charterer and a charteree, depending on the specific need. Fleets can also be temporarily scaled down by *laying-up* ships, which consists of stopping ships at port for a period of time, paying port fees but reducing operating expenses such as manning, storages, and, possibly, insurance.

The utilization of the ships depends on the shipping company's operation mode (see, e.g., Lawrence (1972) and Christiansen et al. (2007)). In what follows we focus, without much loss of generality (see Pantuso et al. (2015a)) on liner shipping operations. Liner shipping companies *deploy* their fleets on a number of *trades*. A trade is a sequence of origin and destination ports in different geographic areas (e.g., Europe to U.S. and Asia to Europe). A ship deployed (i.e., assigned) to a trade (*servicing* the trade) visits some/all of the ports on the trade according to a pre-published schedule, picking up cargoes at origin ports and delivering cargoes at destination ports. Concluded the sailing on one trade, the ship is deployed on another/the same trade with, possibly, some empty (*ballast*) sailing to reposition the ship. Trades are separated into *contractual* and *optional* trades. On contractual trades the shipping company has contractual transportation agreements to be fulfilled while on optional trades no contractual agreement exists. However, the company may, as a strategic decision, choose to start servicing optional trades at any time in the future. This usually corresponds to a long-term commitment equivalent to entering a new market. For most trades the company may wish to ensure a certain number of services per year (frequency) in order to establish a presence in a given market or to satisfy customers requirements.

3   Mathematical Models

In this section, after discussing specific modeling assumptions in Section 3.1, and introducing the notation in Section 3.2, we propose two alternative mathematical models for the MFRP. In Section 3.3 we introduce a model for the maximization of the AIRR. Since the model in Section 3.3 is a mixed-integer nonlinear stochastic program in Section 3.4 we show how the model can be linearized to ease computation. Finally, in Section 3.5 we introduce a profit NPV maximization model.

3.1 Modeling Assumptions

The mathematical models presented in Sections 3.3 and 3.5 have been adapted from the cost minimization model presented in Pantuso et al. (2015a), where the reader can find specific details. We assume our models are tailored for liner shipping of rolling equipment. However, the models do not lose (much) generality, as they can be readily used for (or adapted to) different maritime transportation modes and types of cargoes, as explained in Pantuso et al. (2015a). Here we mention a few elements necessary for introducing the models.

Trades are organized in *loops*. A loop is an ordered sequence of trades. In the model, ships are assigned to loops. Assigning a ship to a loop corresponds to having the ship servicing the trades of the loop in the specified order. Ballast sailing between the trades in the loop (empty ship repositioning) is possible and accounted for in the duration of loop. No transhipment (i.e., movement of cargoes from a ship to another) is considered. Given its strategic relevance, we assume that if the shipping company chooses to service an optional trade in a period, it must continue to service the trade for the rest of the planning horizon. We also assume that different types of cargoes (rolling equipment) need to be transported, as is typical in this shipping segment. Therefore, ships have a capacity (maximum allowance) for each cargo type as well as a total capacity which must be respected. Examples will be given for the specific case study in Section 4.1.

The cost of the capital necessary for buying a ship is, in general, affected by the way the company chooses to finance the ship. Alternative financing decisions (see Stopford (2009) for an overview) will not be considered. Rather, the cost of the capital is included in the costs of the ship. When a ship is built/bought, we consider one unique payment at purchase time. This can in practice represent both the actualized sum of future installments (e.g., debt repayment and interests) and an actual (less likely) upfront cash payment, or a combination of these. Similarly, when a ship is sold/scrapped we assume the company receives one unique payment at purchase time.

When a ship is bought/sold in the second-hand market or scrapped it joins/leaves the fleet at the beginning of the following period. Newbuildings become available after a number of periods representing the lead time from order to delivery. Time charters are available immediately. Ships cannot be sold or scrapped before they are actually delivered. Furthermore,

it is assumed that ships have a set lifetime (typically 20 to 30 years) after which they will leave the fleet. This is common policy for many shipping companies.

We assume most of the parameters of the problem are uncertain. Particularly, the uncertain parameters are: ship values, newbuilding and second-hand prices, selling and scrapping revenues, time chartering rates, space chartering rates, demands, variable operating costs (e.g., bunkering and port fees), the number of ships which can be purchased and sold in the second-hand market, the number of ships which can be chartered in and out. Finally, we assume that a complete representation of the uncertainty is available in the form of a scenario tree (see, e.g., King and Wallace (2012, Ch.4) for an introduction on scenario generation and Pantuso et al. (2015b) for errors related to poor assumptions regarding the scenario tree).

3.2 Notation

In this section we describe the notation used to define the mathematical models. The notation is also reported in tabular form in Appendix A for the reader's convenience.

Let $\mathbf{T} = \{0, ..., \bar{T}\}$ be the set of time periods in the planning horizon and $\mathbf{S}$ the set of scenarios where a scenario is a complete realization of the random parameters for the whole planning horizon. Let $\mathbf{V}_t$ be the set of ships available in the market in period t, and $\mathbf{V}_t^{\mathbf{N}} \subseteq \mathbf{V}_t$ the set of newbuildings which can be delivered in period t. Ships belong to sets $\mathbf{V}_t$ as long as they have not reached their age limit. Let $\mathbf{N}_t$ be the set of all trades that the shipping company may operate in period t, $\mathbf{N}_t^{\mathbf{C}} \subseteq \mathbf{N}_t$ the set of contractual trades and $\mathbf{N}_t^{\mathbf{O}} \subseteq \mathbf{N}_t$ the set of optional trades. Let $\mathbf{L}_t$ be the set of all loops, $\mathbf{L}_{vt} \subseteq \mathbf{L}_t$ the set of loops which can be sailed by a ship of type v in period t, and $\mathbf{L}_{ivt} \subseteq \mathbf{L}_t$ the set of loops that include trade i and which can be sailed by ship v in period t. Note that ships may be forbidden from sailing loops due to, e.g., port restrictions and canal restrictions. Finally, let $\mathbf{K}$ be the set of all cargo types.

As far as decision variables are concerned, given a ship type v, a time period t, and a scenario s, let y_{vts}^P be the number of ships in the fleet, y_{vts}^{SC} the number of ships scrapped, y_{vts}^{NB} the number of ships built, y_{vts}^{SE} the number of ships sold, and y_{vts}^{SH} the number of ships bought in the second-hand market. Let h_{vts}^I and h_{vts}^O be the number of ships chartered in and out, respectively, for one period, where fractions indicate the portion of the period the

8

ship has been chartered for, e.g. 2.5 indicates the charter of two ships for one period and one ship for half of a period. Similarly, let l_{vts}^{U} be the number of ships on lay-up for one period, where fractions indicate the portion of the period ships have been laid-up for. Let x_{vlts} be the number of times loop l is sailed by ships of type v in period t. Let h_{kits}^{S} be the amount of cargo of type k transported by space charters on trade i in period t and scenario s, where *space charters* consist of paying another company for transporting excess cargo. Let δ_{its} be a binary variable indicating whether the company in period t, scenario s, decides to service optional trade i or not. Finally, let variable c_{ts}^{E} denote the capital employed by the shipping company in period t, scenario s.

The model contains the following parameters. The probability of scenario s is p_s. Given a ship type v, a time period t, and a scenario s, let C_{vts}^{NB} be the cost for building a new ship, T^{L} the lead time between order placement and delivery, C_{vts}^{SH} the cost of a ship in the second-hand market, R_{vts}^{SE} the revenue from selling a ship in the second-hand market, and R_{vts}^{SC} the scrapping revenue. Let then R_{vts}^{CO} and C_{vts}^{CI} be the revenue for chartering out and the cost for chartering in, respectively, a ship for one period. Finally, let R_{vs}^{FV} be the value of a ship at the end of the planning horizon ($t = \bar{T}$). As far as operating expenses are considered, let C_{vt}^{OP} be the fixed operating expenses met for a ship of type v in period t (e.g., manning, storages, and insurance). Notice that such expenses are considered deterministic as they are somewhat more controllable or easier to predict. Let R_{vt}^{LU} represents the fixed operating expenses saving obtained when a ship of type v is laid-up in period t. Let then C_{vlts}^{TR} represents the cost of sailing one time loop l with ship type v, in period t under scenario s. Let R_{its}^{D} be the revenue obtained when meeting the transportation demand on trade i, in period t, scenario s, and C_{kits}^{SP} be the cost incurred when delivering one unit of cargo k on space charters on trade t, in time t, scenario s. Let C_{0}^{E} be the value of the fleet at the beginning of the planning horizon, and β the yearly depreciation of the fleet, i.e., the loss of value of the fleet due to ageing. All monetary values are to be assumed appropriately discounted.

Furthermore, given a ship type v, let Y_{vt}^{NB} be the number of ships ordered before the beginning of the planning horizon and delivered in period t, Y_{v}^{IP} the initial number of ships in the pool, and Y_{vts}^{SH} and Y_{vts}^{SE} the maximum number of second-hand purchases and sales, respectively, available in period t, scenario s. Let then Y_{ts}^{SH} and Y_{ts}^{SE} be the maximum number of second-hand purchases and sales, respectively, the company is willing to issue in period t,

1 scenario s. Similarly, let H_{vts}^I and H_{vts}^O be the number of ships of type v which is possible to

2 charter in and out, respectively, for the whole period t, under scenario s, and H_{ts}^I and H_{ts}^O be

3 the total number of ships the company is willing to charter in and out, respectively, in period

4 t, under scenario s. Let then Q_v be the total capacity and Q_{kv} the capacity relative to cargo

5 type k for ships of type v. Let Z_{lv} be the time necessary to complete a loop l with ships of

6 type v and Z_v the fraction of a time period a ship of type v is available. Finally, given a trade

7 i and a time period t, let F_{it} be the minimum number of times the trade must be visited and

8 D_{kits} the amount of cargo type k that must be transported under scenario s.

3.3 Return Maximization Model

10 [Magni](2010), defining the AIRR, show that it can be expressed as the ratio between the

11 present value of a stream of returns and the present value of a stream of investments. Conse-

12 quently, for our scope, we define the AIRR for the renewal of shipping capacity as the ratio

13 between the present value of the stream of profits generated by the shipping services per-

14 formed and the present value of the stream of capital employments. Let $\Pi_{ts}(\psi)$, $t \in \mathbf{T}, s \in \mathbf{S}$

15 be the stream of scenario-dependent one-period profits as a function of ψ, the collection of

16 decision variables (see Section 3.2). Let the operator $PV[\cdot]$ represent the present value of a

17 future amount of money. Finally, let c_{ts}^E, $t \in \mathbf{T}, s \in \mathbf{S}$ be the stream of scenario-dependent

18 capital employments. Our model for the maximization of the expected AIRR (RMax in what

19 follows) can be implicitly expressed as:

$$\max_{\psi \in \Psi} \sum_{s \in \mathbf{S}} (p_s AIRR_s) = \max_{\psi \in \Psi} \sum_{s \in \mathbf{S}} \left(p_s \frac{\sum_{t \in \mathbf{T}} PV[\Pi_{ts}(\psi)]}{\sum_{t \in \mathbf{T}} PV[c_{ts}^E]} \right) \tag{1}$$

20 where $\mathbf{\Psi}$ represents the set of feasible solutions. In what follows the model is introduced

21 explicitly.

The objective function of problem (1) can be explicitly expressed as follows:

$$\max \sum_{s \in \mathbf{S}} \Bigg\{ p_s \frac{1}{\sum_{t \in \mathbf{T}} c_{ts}^E / (\bar{T} + 1)} \tag{2a}$$

$$\Bigg[- \sum_{\substack{t \in \mathbf{T}: \\ t \leq \bar{T} - T^L}} \sum_{v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}} C_{vts}^{NB} y_{vts}^{NB} \tag{2b}$$

$$+ \sum_{\substack{t \in \mathbf{T}: \, v \in \mathbf{V}_t \\ t < \bar{T}}} \left(-C_{vts}^{SH} y_{vts}^{SH} + R_{vts}^{SE} y_{vts}^{SE} + R_{vts}^{SC} y_{vts}^{SC} \right) \tag{2c}$$

$$- \sum_{\substack{t \in \mathbf{T}: \, v \in \mathbf{V}_t \\ t > 0}} \left(C_{vt}^{OP} y_{vts}^P + \sum_{l \in \mathbf{L_{vt}}} C_{vlts}^{TR} x_{vlts} - R_{vt}^{LU} l_{vts}^U \right. \tag{2d}$$

$$\left. + R_{vts}^{CO} h_{vts}^O - C_{vts}^{CI} h_{vts}^I \right) \tag{2e}$$

$$+ \sum_{\substack{t \in \mathbf{T}: \, i \in \mathbf{N}_t^{\mathbf{O}} \\ t > 0}} R_{its}^D \delta_{its} + \sum_{\substack{t \in \mathbf{T}: \, i \in \mathbf{N}_t^{\mathbf{C}} \\ t > 0}} \left(R_{its}^D - \sum_{k \in \mathbf{K}} C_{kits}^{SP} h_{kits}^S \right) \tag{2f}$$

$$+ \sum_{v \in \mathbf{V}_{\bar{T}}} R_{vs}^{FV} y_{v\bar{T}s}^P \Bigg] \Bigg\} \tag{2g}$$

1 Expression (2a) defines the denominator of the expected AIRR, which sums up the present
2 value of future capital employments. Expression (2b) represents the expenses for building
3 new ships. Notice that, in order for a ship to be delivered in period t, it must be ordered
4 T^L period in advance. In (2c) the expenses for buying second-hand ships are summed to the
5 revenue for selling and scrapping ships. Expression (2d) sums up fixed operating expenses
6 (less lay-up savings) and variable operating expenses (i.e., sailing related expenses). In (2e)
7 the revenue for chartering ships out and the expenses for chartering ships in are accounted
8 for. Expression (2f) contains the revenue obtained for transporting cargoes, minus the cost
9 from delivering cargoes by space charters. Finally, (2g) represents the value of the fleet at
10 the end of the planning horizon. Notice that a) (dis)investment decisions such as buying and
11 selling ships are made from period $t = 0$, b) revenues are generated from period $t = 1$ on, as a
12 consequence of previous (dis)investment decisions, c) operating expenses such as chartering,
13 fixed and variable operating costs, are not accounted for in the first time period ($t = 0$) as

11

the initial fleet is the result of past fleet renewal decisions, d) both the numerator and the denominator are to be considered present values as the monetary values such as ship prices and chartering rates are already discounted, e) the objective function represents the expected AIRR as it is the sum of the scenario-AIRR weighed by their probabilities.

The problem is subject to the following constraints.

$$
c_{0s}^E = \beta C_0^E + \sum_{v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}} C_{v0s}^{NB} y_{v0s}^{NB}
$$
$$
+ \sum_{v \in \mathbf{V}_t} (C_{v0s}^{SH} y_{v0s}^{SH} - R_{v0s}^{SE} y_{v0s}^{SE} - R_{v0s}^{SC} y_{v0s}^{SC}), \qquad\qquad s \in \mathbf{S}, \quad (3)
$$

$$
c_{ts}^E = \beta c_{t-1,s}^E + \sum_{v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}} C_{vts}^{NB} y_{vts}^{NB}
$$
$$
+ \sum_{v \in \mathbf{V}_t} (C_{vts}^{SH} y_{vts}^{SH} - R_{vts}^{SE} y_{vts}^{SE} - R_{vts}^{SC} y_{vts}^{SC}), \qquad 1 \le t \le \bar{T} - T^L, s \in \mathbf{S}, \quad (4)
$$

$$
c_{ts}^E = \beta c_{t-1,s}^E + \sum_{v \in \mathbf{V}_t} (C_{vts}^{SH} y_{vts}^{SH} - R_{vts}^{SE} y_{vts}^{SE} - R_{vts}^{SC} y_{vts}^{SC}), \quad \bar{T} - T^L < t \le \bar{T} - 1, s \in \mathbf{S}, \quad (5)
$$

$$
c_{\bar{T}s}^E = \beta c_{\bar{T}-1,s}^E, \qquad\qquad s \in \mathbf{S}. \quad (6)
$$

Constraints (3)-(6) define the value of the capital employed as the sum of the investments in new or second-hand ships, minus the revenues from selling or scrapping available ships. Particularly, the capital employed is defined by (3) for the first period ($t = 0$), by (4) for the periods when it is possible to build new ships, by (5) for the periods when it is not possible to build new ships, and finally by (6) for the last time period ($t = \bar{T}$). Notice that the value of the fleet is depreciated after each period. However, despite the depreciation the value of the fleet may grow with time in case of uprising market conditions.

$$
y_{vts}^P = y_{v,t-1,s}^P + y_{v,t-1,s}^{SH} - y_{v,t-1,s}^{SE} - y_{v,t-1,s}^{SC}, \qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t \setminus \mathbf{V}_t^{\mathbf{N}}, s \in \mathbf{S}, \quad (7)
$$

12

$$y_{vts}^P = Y_{vt}^{NB}, \qquad\qquad t \in \mathbf{T} : t < T^L, v \in \mathbf{V}_t^{\mathbf{N}}, s \in \mathbf{S}, \tag{8}$$

$$y_{vts}^P = y_{v,t-T^L,s}^{NB}, \qquad\qquad t \in \mathbf{T} : t \geq T^L, v \in \mathbf{V}_t^{\mathbf{N}}, s \in \mathbf{S}, \tag{9}$$

$$y_{v0s}^P = Y_v^{IP}, \qquad\qquad v \in \mathbf{V}_0 \setminus \mathbf{V}_0^{\mathbf{N}}, s \in \mathbf{S}, \tag{10}$$

$$y_{vts}^P = y_{vts}^{SC}, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t \setminus \mathbf{V}_{t+1}, s \in \mathbf{S}, \tag{11}$$

$$y_{vts}^{SH} \leq Y_{vts}^{SH}, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{12}$$

$$y_{vts}^{SE} \leq Y_{vts}^{SE}, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{13}$$

$$\sum_{v \in \mathbf{V}_t} y_{vts}^{SH} \leq Y_{ts}^{SH}, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, s \in \mathbf{S}, \tag{14}$$

$$\sum_{v \in \mathbf{V}_t} y_{vts}^{SE} \leq Y_{ts}^{SE}, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, s \in \mathbf{S}. \tag{15}$$

Constraints (7) control the balance of ships bought and sold in the second-hand market or scrapped. Constraints (8) ensure that new buildings ordered before the beginning of the planning horizon are delivered while constraints (9) ensure the delivery of newbuildings within the planning horizon. Constraints (10) set up the initial number of ships of each type in the fleet. Constraints (11) state that ships reaching their age limit must be scrapped. Constraints (12) and (13) set the limit to the number of purchases and sales possible in the second-hand market, respectively. In some circumstances shipping companies may set a limit on the number of second-hand ships they are willing to trade (e.g., they might impose a certain quota of new ships). In this case, constraints (14) and (15) limit the total number of second-hand purchases and sales, respectively.

$$l_{vts} - h_{vts}^I + h_{vts}^O \leq y_{vts}^P, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{16}$$

$$h_{vts}^I \leq H_{vts}^I, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{17}$$

$$h_{vts}^O \leq H_{vts}^O, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{18}$$

$$\sum_{v \in \mathbf{V}_t} h_{vts}^I \leq H_{ts}^I, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, s \in \mathbf{S}, \tag{19}$$

$$\sum_{v \in \mathbf{V_t}} h_{vts}^O \leq H_{ts}^O, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, s \in \mathbf{S}. \tag{20}$$

Constraints (16) state that the number of ships on lay-up or chartered out must actually be available in the fleet. Constraints (17) and (18) set the limit to the number of ships of a given

13

type that is possible to charter in and out, respectively, while constraints (19) and (20) limit the total number of ships the company is willing to charter in or out, respectively, in a time period.

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_{kv} x_{vlts} + h_{kits}^S \geq D_{kits}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^\mathbf{C}, k \in \mathbf{K}, s \in \mathbf{S}, \quad (21)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_{kv} x_{vlts} \geq D_{kits} \delta_{its}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^\mathbf{O}, k \in \mathbf{K}, s \in \mathbf{S}. \quad (22)$$

Constraints (21) and (22) make sure that the demand for each cargo type, on each trade, is satisfied for the contractual and optional trades, respectively. Notice, in constraints (22), that the demand on optional trades must be satisfied only if the company chooses to enter the trade.

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_v x_{vlts} \geq \sum_{k \in \mathbf{K}} (D_{kits} - h_{kits}^S), \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^\mathbf{C}, s \in \mathbf{S}, \quad (23)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_v x_{vlts} \geq \sum_{k \in \mathbf{K}} D_{kits} \delta_{its}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^\mathbf{O}, s \in \mathbf{S}. \quad (24)$$

Constraints (23) and (24) ensure that the total capacity of the ship is not violated when servicing contractual and optional trades, respectively. That is, they ensure that ships do not carry cargo in excess to their capacity.

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} x_{vlts} \geq F_{it}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^\mathbf{C}, s \in \mathbf{S}, \quad (25)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} x_{vlts} \geq F_{it} \delta_{its}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^\mathbf{O}, s \in \mathbf{S}. \quad (26)$$

Constraints (25) and (26) impose frequency requirements on the trades, if they exist. That is, they impose that each trade i is serviced at least a F_{it} times in each period.

$$\sum_{l \in \mathbf{L}_{vt}} Z_{lv} x_{vlts} \leq Z_v (y_{vts}^P + h_{vt}^I - h_{vt}^O - l_{vts}^U), \qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}. \quad (27)$$

14

Constraints (27) state that, for a given ship type, the total sailing time should not exceed the total time available for that ship type. As an example, if a ship is available for 2/3 of a period (due, e.g., to maintenance), the total sailing of the ship cannot exceed 2/3 of a period.

$$\delta_{its} \leq \delta_{i,t+1,s}, \qquad\qquad t \in \mathbf{T} \setminus \{0, \bar{T}\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}. \qquad (28)$$

Constraints (28) ensure that when the company chooses to service an optional trade, the trade is serviced for the rest of the planning horizon. The choice of entering a new trade is in fact considered a strategic decisions which impacts a number of planning periods.

$$y_{vts}^{NB} \in \mathbf{Z}^+, \qquad\qquad t \in \mathbf{T} : t \leq \bar{T} - T^L, v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}, s \in \mathbf{S}, \qquad (29)$$

$$y_{vts}^{SC} \in \mathbf{Z}^+, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (30)$$

$$y_{vts}^{SH} \in \mathbf{Z}^+, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (31)$$

$$y_{vts}^{SE} \in \mathbf{Z}^+, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (32)$$

$$y_{vts}^{P} \in \mathbf{R}^+, \qquad\qquad t \in \mathbf{T}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (33)$$

$$l_{vts}^{U} \in \mathbf{R}^+, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (34)$$

$$h_{vts}^{I} \in \mathbf{R}^+, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (35)$$

$$h_{vts}^{O} \in \mathbf{R}^+, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (36)$$

$$x_{vlts} \in \mathbf{R}^+, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, l \in \mathbf{L}_{vt}, s \in \mathbf{S}, \qquad (37)$$

$$h_{kits}^{S} \in \mathbf{R}^+, \qquad\qquad k \in \mathbf{K}, t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{C}}, s \in \mathbf{S}, \qquad (38)$$

$$c_{ts}^{E} \in \mathbf{R}^+, \qquad\qquad t \in \mathbf{T}, s \in \mathbf{S}, \qquad (39)$$

$$\delta_{its} \in \{0, 1\}, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}. \qquad (40)$$

Finally, constraints (29)-(40) set the domain for each decision variable. Model RMax (2)-(40) is to be considered nonanticipative, and nonanticipativity constraints define wether the model is two-stage or multistage. However, nonanticipativity constraints are not shown for the sake of legibility.

3.4 Linearization of the AIRR Maximization Model

The RMax model presented in Section 3.3 is a nonlinear mixed-integer (possibly multistage) stochastic program. Objective function (2) is a linear-fractional function of the decision variables. In order to ease the solution process we propose a linearization of the RMax model based on Charnes and Cooper (1962).

Let us define a new decision variable as follows:

$$w = \frac{\bar{T} + 1}{\sum\limits_{t \in \mathbf{T}} \sum\limits_{s \in \mathbf{S}} p_s c_{ts}^E} \tag{41}$$

Decision variable w has no economic meaning. It is a mere mathematical artefact by which it is possible to create new variables matching the variables defined for the RMax model. The utilization of these new variables, in substitution or in addition to the original variables depending on the case, allows writing a linear model equivalent to the RMax model. The new variables inherit the name of the variables they match, with the addition of the bar accent $(\bar{\cdot})$.

For continuous variables new variables are defined as the product of w and the original variable. As an example, variable $\bar{x}_{vlts}$, matching x_{vlts} is defined as:

$$\bar{x}_{vlts} = w x_{vlts}, \quad \forall t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, l \in \mathbf{L}_{vt}, s \in \mathbf{S}. \tag{42}$$

For all other continuous variables of the RMax model new variables are created in the same way, and replace the original continuous variables in the resulting linearized model.

For binary variables δ_{it} we adopt the method described by Glover (1975). We need to create the following relationship:

$$\bar{\bar{\delta}}_{its} = w \delta_{its}, t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}. \tag{43}$$

However, since δ_{its} is binary, we need to ensure that $\bar{\bar{\delta}}_{its}$ takes either value w or 0. Therefore, $\bar{\bar{\delta}}_{its}$ cannot directly replace variables δ_{its}. Instead, relationship (43) will be ensured by adding constraints (44)-(46) in the linearized model.

$$w - \bar{\bar{\delta}}_{its} + U \delta_{its} \leq U, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}, \tag{44}$$

$$\bar{\bar{\delta}}_{its} - w \leq 0, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}, \tag{45}$$

$$\bar{\bar{\delta}}_{its} - U \delta_{its} \leq 0 \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}. \tag{46}$$

where U is an upper bound on the value of w. This means that the linearized model will contain both $\bar{\delta}_{its}$ and δ_{its} variables.

General integer variables (i.e., y_{vts}^{NB}, y_{vts}^{SC}, y_{vts}^{SH} and y_{vts}^{SE}) must be transformed into binary variables before using the method described by Glover (1975). Several alternatives are available in order to transform general integer variables into binary variables. Since a complete examination is beyond the scope of this paper, in what follows we describe the transformation which performed best for the case study presented in Section 4.

Assume that for each ship type a decision is made about whether or not to order an individual ship of that type. Let $\mathbf{J^{NB}}$ be the set of such decisions. Correspondingly, $|\mathbf{J^{NB}}|$ is the maximum number of ships which is possible to order. Let then y_{jvts}^{NB} be a binary variable indicating whether the j-th ship is ordered or not. The correspondence to the original y_{vts}^{NB} variables is the following:

$$y_{vts}^{NB} = \sum_{j \in \mathbf{J^{NB}}} y_{jvts}^{NB}, \quad \forall t \in \mathbf{T} : t \leq \bar{T} - T^L, v \in \mathbf{V^N_{t+T^L}}, s \in \mathbf{S}. \tag{47}$$

Similarly, variables y_{vts}^{SC}, y_{vts}^{SH} and y_{vts}^{SE} are transformed into the corresponding y_{jvts}^{SC}, y_{jvts}^{SH} and y_{jvts}^{SE}, and the associated sets $\mathbf{J^{SC}}$, $\mathbf{J^{SH}}$, and $\mathbf{J^{SE}}$ are created. Once binary variables are obtained, the corresponding $\bar{y}_{jvts}^{NB}$, $\bar{y}_{jvts}^{SC}$, $\bar{y}_{jvts}^{SH}$, and $\bar{y}_{jvts}^{SE}$ can be created using the relationships in Glover (1975), as shown for the δ_{its} variables.

However, this formulation leads to symmetry problems. Ships of the same type are identical (i.e., they have the same cost and technical features). Therefore, ordering (scrapping, buying, selling) ship j of type v, is identical to ordering (scrapping, buying, selling) ship $j + 1$. Symmetry problems can be tackled in several ways, and also in this case an exhaustive examination is beyond the scope of the paper. The solution we adopted consists of adding constraints of type (48) which ensure that if m ships are to be built, the first m variables $y_{1vts}^{NB}, \ldots, y_{mvts}^{NB}$ take value one, and zero the remaining.

$$y_{jvts}^{NB} \leq y_{j-1,v,t,s}^{NB}, \quad j \in \mathbf{J^{NB}} \setminus \{1\}, t \in \mathbf{T} : t \leq \bar{T} - T^L, v \in \mathbf{V^N_{t+T^L}}, s \in \mathbf{S}. \tag{48}$$

This is equivalent to selecting any other combinations of m indices from $\mathbf{J^{NB}}$. Similar constraints have been added for variables y_{jvts}^{SH}, y_{jvts}^{SE}, and y_{jvts}^{SC}.

The full linearized version of RMax model is reported in Appendix B for the sake of legibility. Mathematical model (51)-(112) is a linear mixed integer stochastic program equivalent

17

to the RMax model and is, in general, easier to solve. Once the linearized model has been solved, the values of the continuous variables of the original model can be obtained through relationships of type (42), the values of the general integer variables can be obtained through relationships of type (47), while the values of variables δ_{its} is part of the solution to the linearized model.

3.5 Profit Maximization Model

The profit maximization (PMax) model consists of selecting a fleet renewal plan which maximizes the expected NPV of future cash flows, corresponding to the present value of future profits. The PMax model is hence:

$$\max_{\psi' \in \Psi'} \sum_{s \in \mathbf{S}} \left(p_s \sum_{t \in \mathbf{T}} PV[\Pi_{ts}(\psi')] \right) \tag{49}$$

where $\psi' \in \Psi'$ is the collection of decision variables. Notice that ψ' is different than ψ defined in Section 3.3 as the PMax model does not contain capital employment variables c_{ts}^E.

PMax model can be explicitly defined as follows:

$$\max \sum_{s \in \mathbf{S}} \left\{ p_s \left[- \sum_{\substack{t \in \mathbf{T}: \\ t \leq \bar{T} - T^L}} \sum_{v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}} C_{vts}^{NB} y_{vts}^{NB} \right. \right. \tag{50a}$$

$$+ \sum_{\substack{t \in \mathbf{T}: \, v \in \mathbf{V}_t \\ t < \bar{T}}} \left(-C_{vts}^{SH} y_{vts}^{SH} + R_{vts}^{SE} y_{vts}^{SE} + R_{vts}^{SC} y_{vts}^{SC} \right) \tag{50b}$$

$$- \sum_{\substack{t \in \mathbf{T}: \, v \in \mathbf{V}_t \\ t > 0}} \left(C_{vt}^{OP} y_{vts}^P + \sum_{l \in \mathbf{L_{vt}}} C_{vlts}^{TR} x_{vlts} - R_{vt}^{LU} l_{vts}^U \right. \tag{50c}$$

$$\left. + R_{vts}^{CO} h_{vts}^O - C_{vts}^{CI} h_{vts}^I \right) \tag{50d}$$

$$+ \sum_{\substack{t \in \mathbf{T}: \, i \in \mathbf{N}_t^{\mathbf{O}} \\ t > 0}} R_{its}^D \delta_{its} + \sum_{\substack{t \in \mathbf{T}: \, i \in \mathbf{N}_t^{\mathbf{C}} \\ t > 0}} \left(R_{its}^D - \sum_{k \in \mathbf{K}} C_{kits}^{SP} h_{kits}^S \right) \tag{50e}$$

$$\left. \left. + \sum_{v \in \mathbf{V}_{\bar{T}}} R_{vs}^{FV} y_{v\bar{T}s}^P \right] \right\} \tag{50f}$$

subject to $(7) - (40)$

18

Notice that PMax model contains the same constraints as in (2)-(40), except for constraints (3)-(6) defining the capital employed. Notice also that objective function (50) represents the present value of a stream of profits. In fact, all the monetary values are implicitly actualized.

4 Comparing Maritime Fleet Renewal Problem Models

In this section we compare the RMax and the PMax models introduced in Section 3. In Section 4.1 we introduce a set of instances based on the case of a major liner shipping company transporting rolling equipment. In Section 4.2 we propose a comparison based on the results both in terms of AIRR and profit obtained with the two cases, as well as a discussion on the solutions obtained.

The models were implemented using Xpress Mosel modeling language, and solved using Xpress Optimizer Version 24.01.04 on an Intel$^®$ Core^{TM} i7-3770 CPU @ 3.4 GHz machine with 16 GB RAM. As far as the RMax model is concerned, all tests were done solving its linearized version illustrated in Section 3.4 and shown in full in Appendix B.

4.1 Case Study and Instances

Instances have been generated using data from a major liner shipping company engaged in the transportation of rolling equipment. The company transports three main types of cargoes, namely cars, high and heavy vehicles (HH) and *breakbulk* cargo (BB). The ships operated belong to three main families, namely Pure Car Carriers (PCCs), specialized for cars, Pure/Large Car Truck Carrier (PCTC/LCTCs), which can carry a mix of cars and trucks, and Roll On-Roll Off (RORO) ships which can carry almost any combination of the three types of cargoes. Each ship has a specified capacity of each type of cargo, as well as a total capacity to respect. The company operates several trades around the world. An internal policy at the company, combined with operating in a very specialized segment, imposes that only new ships are purchased, and that the ships the company owns are kept until they are scrapped. Charters in and out are however possible. The models presented in Section 3 have been slightly modified accordingly (i.e., a zero-upper-bound has been imposed on second-hand purchases and sales).

Three instances have been generated, namely Small (S), Medium (M), and Large (L), de-

Problem	Instance	# Variables (Integer)	# Constraints
	S	$\approx$ 18 000 (1 200)	$\approx$ 3 000
PMax	M	$\approx$ 40 000 (1 600)	$\approx$ 4 000
	L	$\approx$ 75 000 (1 800)	$\approx$ 5 000
	S	$\approx$ 30 000 (15 000)	$\approx$ 65 000
RMax	M	$\approx$ 55 000 (19 000)	$\approx$ 80 000
	L	$\approx$ 95 000 (23 000)	$\approx$ 95 000

Table 1: Size of the problems.

pending on the number of ship types and trades included. Ships with identical characteristics (same family) but different age are treated as different ship types. Instance L mimics a large liner shipping company such as the focal company, with an initial fleet of 55 ships, servicing 11 to 14 trades – three trades are optional. Instance M describes a smaller shipping company with an initial fleet of 35 ships, servicing 7 to 11 trades. Finally, instance S corresponds to a shipping company with an initial fleet of 27 ships, servicing 5 to 8 trades. Each instance includes optional trades to represent opportunities of expansion into new markets. Tables 6 and 7 in Appendix C report the ship types and trades included in each instance, while Table 1 reports the size of problems. As it can be observed, the linearization of the RMax model generates a dramatic increase in the number of constraints and integer variables. The increase in the number of constraints is mainly due to the introduction of the relationship between the original binary variables and the linearization variables (75)-(90). The additional integer variables are instead generated by the need of transforming the original integer variables in binary variables.

The planning horizon has been set to five years, in accordance with the length of the forecast at the company. All economic parameters, such as ship prices, charter rates, and operating expenses have been estimated using raw data from the focal company. All input values have been properly discounted over the planning horizon, using a discount factor of 12%, as suggested in Stopford (2009).

In order to account for uncertainty a two-stage scenario tree has been generated, with the current period representing the first-stage, and the second-stage representing the following five years. Figure 1 reports a qualitative description of the scenario tree. Three random variables
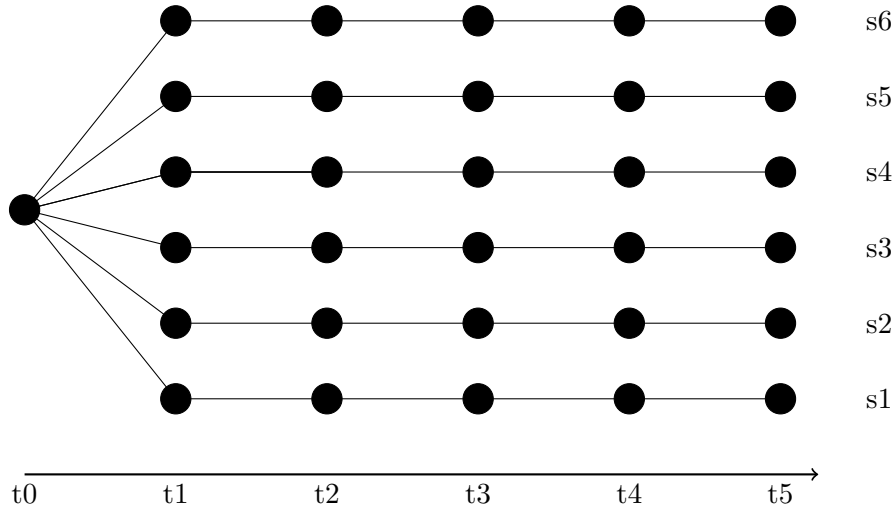
20

Figure 1: Qualitative description of a two-stage scenario tree with six scenarios.

have been introduced, namely, *market status*, *steel price*, and *fuel price*. The market status variable controls freight revenue, demand, and newbuilding prices, and charter rates. Steel prices impact the scrapping revenue, whilst fuel prices impact sailing costs. A distribution of forecast errors for the three random variables has been estimated based on the data from the case company. Scenarios have been generated using a modified version of the method proposed by Høyland et al. (2003), using distribution functions instead of moments to control the margins. Acceptable in-sample stability (see Kaut and Wallace (2007)) is achieved with six scenarios for both the PMax and RMax models, by accepting a standard deviation of approximately 0.6% of the objective. In fact, this value of standard deviation is at least one order of magnitude smaller than the numerical results we discuss in what follows. All tests have been therefore run with six scenarios and solved to proven 0.5% optimality gap. Each test has been performed by running the models ten times, each time with a different scenario tree.

4.2 Results and Discussion

A comparison between the results obtained with the RMax and PMax models is proposed along the following dimensions: in Table 2 we compare the economic results from the two models, in Table 3 we propose a comparison of the solutions, and finally in Table 4 we

compare the sensitivity to uncertainty through the *value of the stochastic solution* (VSS) (see Birge (1982)).

Table 2 shows the economic results obtained from the two models. For both models the table shows the expected profit (as a fraction of the profit obtained with the PMax model), the expected AIRR (as a fraction of the AIRR obtained with the RMax model), and the expected "Compounded Annual Growth Rate" (CAGR) which measures the period-by-period growth rate of the investment over the whole planning horizon, i.e., how much the investment grows from a year to another, represented by a constant rate.

An expected result is that the PMax model generates a smaller AIRR, and that the RMax model generates a smaller profit than the other model. As seen in Table 2, the RMax model is able to achieve a 13.6-14.9% higher AIRR by giving up 7.0-9.0% of profits. Similarly, the PMax model gives a 7.5-9.9% higher profits but at the price of a 12.0-13.0% smaller AIRR. The results are consistent over all the instances representing companies of different sizes. The extra profit gained with the PMax model is of course of value, but is generated by a higher employment of capital – Table 3 shows the higher number of ships the PMax model suggests investing in. In this situation one may want to know whether it is sound to employ that extra capital in order to gain extra profits.

Useful information in this sense may be found by looking at the CAGR generated by the two models (see Table 2). In both cases the capital employed grows at a rate higher than 20%, with the RMax model ensuring an approximately 2% higher growth. However, the CAGR generated by the extra capital employed in the PMax solution is nearly halved compared to the capital employed in the RMax solution. This illustrates that the capital employed for extra profit is expected to grow at a much slower rate. Whether to employ the extra capital for additional profits would depend on the presence of other viable employment alternatives. Note, however, that the values of CAGR reported in Table 2) cannot be directly compared to actual CAGR values in the maritime industry. The values reported here do not take into account a number of additional expenses actually paid by real shipping companies, such as administrative costs (insurance, cost of the administrative personnel) and taxation. We neglect such additional expenses, because a) they would not play any role in the optimization models presented as they are constants, and b) their precise estimate is made particularly difficult due to the international set up of the focal shipping company. Therefore, we expect

22

the values reported in Table 2) to be in general higher than historical market values and thus,
only useful for a comparison in the scope of this paper.

Table 2: Expected economic results from the RMax and PMax models. Averages over 10 runs.

	RMax			PMax		
	L	M	S	L	M	S
Profit	0.93	0.91	0.91	1	1	1
AIRR	1	1	1	0.87	0.88	0.88
CAGR	24.1 %	23.8 %	23.3 %	21.9 %	21.8 %	21.4 %
CAGR Extra Capital	-	-	-	10.4 %	12.2 %	11.9 %

Table 3 reports the solutions to both the RMax and PMax models. The RMax model consistently (over the instances) suggests building much fewer ships in the first period than the PMax model, and the same trend is to some extent maintained in the following periods. The aggressive investment strategy suggested by the PMax model is rarely seen in practice, and it can be discussed whether it is realistic for a company to make investments of this size, compared to the initial value of the fleet. On the other hand, the slow-paced newbuilding policy suggested by the RMax model is more consistent with common practice in many shipping companies. Hence, the results of the RMax model could appear more intuitive from a shipping company's perspective. Furthermore, investing in as many ships as suggested by the PMax solution (e.g., 19 for the Large instance) raises doubts on whether capital expenses and debt repayments can actually be afforded in day-to-day operations. Investments of this magnitude are rarely seen in the industry. As an example, the focal shipping company typically invests in less than a hand-full of ships every year. The RMax formulation will only choose to invest in a new ship if the investment will lead to an equal or higher return than without the investment, which means that the extra profit gained over the capital needed for the investment must be higher than the return without the investment. If it is only marginally lower, the investment will not be made. For the PMax model it is enough that the extra investment gives a net profit, as long as it is positive. So the RMax model can be said to have a stricter restriction on the profit an investment must generate.

As long as the scrapping policy is concerned, most of the ships scrapped by the PMax

model are the ones reaching the age limit. The RMax model instead suggests scrapping more of the younger ship types in addition to the ones leaving the fleet. These results are consistent with the difference shown for the building policy. In fact, while the PMax model pursues a more aggressive capacity expansion policy, the RMax model seeks renewal also for efficiency purposes. The average scrapping age suggested by the RMax model is close to the average scrapping age in the market (25 years) while the PMax model typically scraps ships when the reach the age limit imposed by the model (30 years).

The number of new trades entered by the PMax model is also consistent with its new building and scrapping policy. Table 3 shows that the expansion suggested by the PMax model is consistently choosing to service all optional trades but one. The trade never chosen is characterized by a relatively low demand and high frequency requirement. Again, the RMax model enters new markets only if the corresponding remuneration increases the return on the capital employment.

Table 3: Solutions to the RMax and PMax models. Averages over 10 runs.

		RMax			PMax		
	Period (t)	L	M	S	L	M	S
Newbuildings	0	6.7	3.6	2.6	19.4	8.6	5.9
	$1, .., \overline{T} - \overline{T}^{L}$	12.1	6.4	4.4	11.9	8.5	5.6
Scrappings	0	-	-	-	-	-	-
	$1, .., \overline{T} - 1$	12.9	8.1	5.6	10.8	7.3	5.2
New Trades	1	0.78	1.18	0.80	0.97	1.67	1.07
	2	0.78	1.18	0.80	1.65	2.50	1.57
	3	0.80	1.20	0.82	2.00	3.00	2.00
	4	0.80	1.20	0.82	2.00	3.00	2.00
	5	0.83	1.22	0.87	2.00	3.00	2.00

Table 4 reports the VSS for the two models. It can be noticed that, except for the large instance, the VSS is higher for the RMax model. It shows that planning against uncertainty has a much higher impact when maximizing the AIRR. The reason for the higher VSS for the RMax model lays mainly in the smaller rate of expansion of the fleet. A smaller fleet is more vulnerable to changes in demand as it offers fewer opportunities to increase the return by improving the management of the fleet. If the solution to the mean value problem for the

24

RMax model was used, it would generate a high amount of space charters to cater for peaks in demand. This is a symptom of tonnage deficit. On the opposite, the mean value solution to the PMax model will suggest building a bigger fleet (compared to the mean value solution to the RMax model). Bigger fleets offer more flexibility when it comes to recovering from changes in demand. Therefore, returns are more affected by uncertainty than profits. This suggests that maximizing the AIRR calls for planning against uncertainty, as plans made with average data are more subject to result in unbalanced fleets.

Table 4: Value of the Stochastic Solution (VSS) for the RMax and PMax models. Averages over 10 runs.

	RMax			PMax		
	L	M	S	L	M	S
VSS	11.9 %	15.9 %	21.7 %	12.6 %	3.3 %	11.7 %

Finally, tests were run slightly modifying the instances introduced earlier in order to validate the results presented. For the sake of conciseness we just summarize our findings. Initially, we compared the two models with an increased freight revenue (parameter R_{its}^D). The freight revenue obtained by the case company was increased (possibly unrealistically) by up to 50%. The rationale behind this is that with a higher revenue for the cargoes transported, also the RMax model might find it profitable to invest more. However, the structural differences in the solutions shown in Table 3 was confirmed. The main difference is in the fact that the PMax model chooses to service all optional trades, and to invest slightly more. However, for the RMax model the increased revenue was not sufficient to enter additional optional trades and expand the fleet. Then, with respect to original instances, we increased the number of optional trades ($|N_t^O|$). Particularly, we expanded instances S and M with two additional optional trades in each. The higher number of optional trades pushed the RMax model to increase the number of newbuildings. However, this result is ever more marked for the PMax model, stressing the structural differences highlighted in the base case. A higher number of optional trades makes the VSS drop for the PMax model, confirming that the availability of more revenue options, together with a bigger fleet, offer decision makers more resilience in case of poor fleet planning decision. The structural difference between the two models was also confirmed when considering a longer planning horizon, i.e., $\bar{T} = 10$. When decreasing

25

the space charter cost (C_{kits}^{SP}), both models suggest investing in fewer ships (though the PMax model still suggests investing more than the RMax model). The option of sending cargoes by space charters is used more, and consequently fewer own ships are needed. Intuitively, the VSS slightly decreases for both models, as space charters represent in this case a rather cheap mean for recovering from poor fleet planning decisions. Finally, we considered three stages rather than two (i.e., we solved three-stage stochastic programs for both models). The idea is that, in a two-stage structure, it may happen that investments are postponed until after the uncertainty is disclosed. However, also with three-stages, the results shown with the two-stage case are, to a very large extent, confirmed.

The adoption of the RMax in the maritime industry could confirm analytically the strategic ideas of shipping investors. In fact, as shown in this section, the solutions provided are to a large extent compatible with common practices in the maritime industry. In addition, since the solutions to the RMax model are balanced against an uncertain future, its adoption may help to prevent poor investment decisions merely caused by feelings induced by the raising or falling of shipping markets.

5 Conclusions

Strategic decisions related to capacity expansions or renewal, besides considering various operating constraints, may require attention to a number of financial indicators, such as the Average Internal Rate of Return (AIRR) besides or in addition to net present values. In this paper we focused on the case of maritime shipping capacity renewal. Shipping is a capital intensive industry where uncertainty plays a major role. In addition shipping capacity is characterized by a long economic life. For these reasons, particular attention must be paid to the financial, in addition to the technical, fitness of investments in maritime shipping capacity.

We proposed a AIRR maximization model for the renewal of maritime shipping capacity, as well as a reformulation to eliminate the nonlinear interaction between the decision variables. The model was compared with a more classic profit maximization model, based on the available literature, on the case of a major liner shipping company. The comparison shows that the profit loss incurred by the AIRR maximization model is smaller than the AIRR loss incurred by the profit maximization model, and that the extra profit generated

26

by the profit maximization model grows at a slower rate. Furthermore, we show that the profit maximization model pursues an aggressive expansion policy, while the solutions offered by the AIRR maximization model are more consistent with common practice in the shipping industry, hence may better represent the preferences of most shipping investors. Particularly, large publicly-traded shipping companies may find it more appropriate to define strategies which maximize the return for their investors. However, none of the models is meant to be preferable. As an example, small family-owned shipping companies may still find it reasonable to maximize profits. In any case, the AIRR maximization model extends the available literature offering companies also the possibility to select capital employment decisions which maximize their return.

Several extensions or complementary analyses could be set up in future research efforts. The choice between profit or return as a metric could be facilitated by a multi-objective model which considers the two objectives. Such a model would provide a Pareto frontier illustrating the trade-offs between the metrics involved, leaving the user decide what combination of them fits better the scope of the company. However, such model would pose additional non-trivial challenges. In an attempt to maximize the weighed sum of the objectives, the way the two objected should be normalized is far from obvious, given the magnitude difference between AIRR and profit, and the high number of variables involved in the model. In addition, evaluating Pareto solutions without some assistance from the shipping industry is not trivial. It would be highly beneficial to receive inputs from the industry on what different combinations of profit and AIRR would mean in practice. Related to this, a potential research avenue is a game-theoretical analysis of the industry, under the assumptions that profits or returns (or a combination of them) were maximized. This would shed light on how the adoption of such analytical tool would impact the whole industry.

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A   Notation

Sets	
$\mathbf{T}$	Set of time periods
$\mathbf{S}$	Set of scenarios
$\mathbf{V}_t$	Set of ships available in the market in period t
$\mathbf{V}_t^{\mathbf{N}} \subseteq \mathbf{V}_t$	Set of newbuildings which can be delivered in period t
$\mathbf{N}_t$	Set of all trades that the shipping company may operate in period t
$\mathbf{N}_t^{\mathbf{C}} \subseteq \mathbf{N}_t$	Set of contractual trades
$\mathbf{N}_t^{\mathbf{O}} \subseteq \mathbf{N}_t$	Set of optional trades
$\mathbf{L}_t$	Set of all loops
$\mathbf{L}_{vt} \subseteq \mathbf{L}_t$	Set of loops which can be sailed by a ship of type v in period t
$\mathbf{L}_{ivt} \subseteq \mathbf{L}_t$	Set of loops which include trade i and can be sailed by ship v in period t
$\mathbf{K}$	Set of all cargo types
Variables	
y_{vts}^{P}	Number of ships of type v in the fleet in period t, scenario s
y_{vts}^{SC}	Number of ships of type v scrapped in period t, scenario s
y_{vts}^{NB}	Number of ships of type v built in period t, scenario s
y_{vts}^{SE}	Number of ships of type v sold in period t, scenario s

y_{vts}^{SH}	Number of ships of type v bought in the second-hand market in period t, scenario s
h_{vts}^{I}	Number of ships of type v chartered-in for a period in period t, scenario s
h_{vts}^{O}	Number of ships of type v chartered-out for a period in period t, scenario s
l_{vts}^{U}	Number of ships of type v on lay-up for a period in period t, scenario s
x_{vlts}	Number of times loop l is sailed by ships of type v in period t, scenario s
h_{kits}^{S}	Amount of cargo of type k transported by space charters on trade i in period t,scenario s
δ_{its}	Binary variable indicating whether optional trade i is serviced or not in period t, scenario s
c_{ts}^{E}	Capital employed by the shipping company in period t, scenario s

Parameters

$\bar{T}$	The last time period in the planning horizon
p_s	The probability of scenario s
C_{vts}^{NB}	The cost for building a new ship of type v in period t, scenario s
T^L	The lead time between order placement and delivery
C_{vts}^{SH}	The cost of a ship of type v in period t, scenario s, in the second-hand market
R_{vts}^{SE}	The revenue from selling a ship of type v in period t, scenario s, in the second-hand market
R_{vts}^{SC}	The revenue from scrapping a ship of type v in period t, scenario s
R_{vts}^{CO}	The revenue from chartering out for a period a ship of type v in period t, scenario s
C_{vts}^{CI}	The cost for chartering in for one period a ship of type v in period t, scenario s
R_{vs}^{FV}	The value of a ship of type v at the end of the planning horizon under scenario s
C_{vt}^{OP}	The fixed operating expenses met for a ship of type v in period t

R_{vt}^{LU}	The fixed operating expenses saving obtained when a ship of type v is laid-up in period t
C_{vlts}^{TR}	The cost of sailing one time loop l with ship type v, in period t,scenario s
R_{its}^{D}	The revenue obtained when meeting the transportation demand on trade i, in period t, scenario s
C_{kits}^{SP}	The cost incurred when delivering one unit of cargo k on space charters on trade t, in time t, scenario s
C_{0}^{E}	The value of the fleet at the beginning of the planning horizon
β	The yearly depreciation of the fleet
Y_{vt}^{NB}	The number of ships of type v ordered before the beginning of the planning horizon and delivered in period t
Y_{v}^{IP}	The initial number of ships of type v in the pool
Y_{vts}^{SH}	The number ships of type v available in the second-hand market in period t, scenario s
Y_{vts}^{SE}	The number ships of type v the company can sell market in period t, scenario s
Y_{ts}^{SH}	The maximum number of second-hand purchases the company is willing to issue in period t, scenario s
Y_{ts}^{SE}	The maximum number of sales the company is willing to issue in period t, scenario s
H_{vts}^{I}	The number of ships of type v which is possible to charter in for the whole period t, scenario s
H_{vts}^{O}	The number of ships of type v which is possible to charter out for the whole period t, scenario s
H_{ts}^{I}	The total number of ships the company is willing to charter out in period t, scenario s
H_{ts}^{O}	The total number of ships the company is willing to charter out in period t, scenario s
Q_{v}	The total capacity of ship type v
Q_{kv}	The capacity of cargo type k of ships of type v

Z_{lv}	The time necessary to complete a loop l with ships of type v
Z_v	The fraction of a time period a ship of type v is available
F_{it}	The minimum number of times trade i must be visited in period t
D_{kits}	The amount of cargo type k that must be transported across trade i in period t, scenario s

B   Linearized AIRR Maximization Model

The RMax model (2b)-(40) can be written in the following linear equivalent way, as explained in Section 3.4. Particularly, for our specific case we empirically estimated that $U = 1$ is a valid upper bound on w for all instances.

$$
\max \Pi = \sum_{s \in \mathbf{S}} p_s \left[- \sum_{\substack{t \in \mathbf{T}: \\ t \leq \bar{T} - T^L}} \sum_{v \in \mathbf{V}^{\mathbf{N}}_{t+T^L}} \sum_{j \in \mathbf{J}^{\mathbf{NB}}} C^{NB}_{vts} \bar{y}^{NB}_{jvts} \right. \tag{51a}
$$

$$
+ \sum_{\substack{t \in \mathbf{T}: \\ t < \bar{T}}} \sum_{v \in \mathbf{V}_t} \left(- \sum_{j \in \mathbf{J}^{\mathbf{SH}}} C^{SH}_{vts} \bar{y}^{SH}_{jvts} + \sum_{j \in \mathbf{J}^{\mathbf{SE}}} R^{SE}_{vts} \bar{y}^{SE}_{jvts} + \sum_{j \in \mathbf{J}^{\mathbf{SC}}} R^{SC}_{vts} \bar{y}^{SC}_{jvts} \right) \tag{51b}
$$

$$
- \sum_{\substack{t \in \mathbf{T}: \\ t > 0}} \sum_{v \in \mathbf{V}_t} \left(C^{OP}_{vt} \bar{y}^P_{vts} + \sum_{l \in \mathbf{L}_{vts}} C^{TR}_{vlts} \bar{x}_{vlts} - R^{LU}_{vt} \bar{l}^U_{vts} \right. \tag{51c}
$$

$$
\left. + R^{CO}_{vts} \bar{h}^O_{vts} - C^{CI}_{vts} \bar{h}^I_{vts} \right) \tag{51d}
$$

$$
+ \sum_{\substack{t \in \mathbf{T}: \\ t > 0}} \sum_{i \in \mathbf{N}^{\mathbf{O}}_t} R^D_{its} \bar{\delta}_{its} + \sum_{\substack{t \in \mathbf{T}: \\ t > 0}} \sum_{i \in \mathbf{N}^{\mathbf{C}}_t} \left(R^D_{its} w - \sum_{k \in \mathbf{K}} C^{SP}_{kits} \bar{h}^S_{kits} \right) \tag{51e}
$$

$$
\left. + \sum_{v \in \mathbf{V}_{\bar{T}}} R^{FV}_{vs} \bar{y}^P_{v\bar{T}s} \right] \tag{51f}
$$

34

1 subject to:

$$
\bar{c}^E_{ts} = \begin{cases}
\beta C^E_0 w + \sum\limits_{v \in \mathbf{V}^{\mathbf{N}}_{t+T^L}} \sum\limits_{j \in \mathbf{J}^{\mathbf{NB}}} C^{NB}_{v0s} \bar{y}^{NB}_{jv0s} \\
\qquad + \sum\limits_{v \in \mathbf{V}_t} \left(\sum\limits_{j \in \mathbf{J}^{\mathbf{SH}}} C^{SH}_{v0s} \bar{y}^{SH}_{jv0s} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SE}}} R^{SE}_{v0s} \bar{y}^{SE}_{jv0s} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SC}}} R^{SC}_{v0s} \bar{y}^{SC}_{jv0s} \right), & t = 0, s \in \mathbf{S}, \\[2ex]
\beta \bar{c}^E_{t-1,s} + \sum\limits_{v \in \mathbf{V}^{\mathbf{N}}_{t+T^L}} \sum\limits_{j \in \mathbf{J}^{\mathbf{NB}}} C^{NB}_{vts} \bar{y}^{NB}_{vts} \\
\qquad + \sum\limits_{v \in \mathbf{V}_t} \left(\sum\limits_{j \in \mathbf{J}^{\mathbf{SH}}} C^{SH}_{vts} \bar{y}^{SH}_{vts} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SE}}} R^{SE}_{vts} \bar{y}^{SE}_{vts} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SC}}} R^{SC}_{vts} \bar{y}^{SC}_{vts} \right), & 1 \leq t \leq \bar{T} - T^L, s \in \mathbf{S}, \\[2ex]
\beta \bar{c}^E_{t-1,s} + \sum\limits_{v \in \mathbf{V}_t} \left(\sum\limits_{j \in \mathbf{J}^{\mathbf{SH}}} C^{SH}_{vts} \bar{y}^{SH}_{vts} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SE}}} R^{SE}_{vts} \bar{y}^{SE}_{vts} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SC}}} R^{SC}_{vts} \bar{y}^{SC}_{vts} \right), & \bar{T} - T^L < t \leq \bar{T} - 1, s \in \mathbf{S}, \\[2ex]
\beta \bar{c}^E_{t-1,s}, & t = \bar{T}, s \in \mathbf{S}.
\end{cases}
\tag{52}
$$

$$
\bar{y}^P_{vts} = \bar{y}^P_{v,t-1,s} + \sum\limits_{j \in \mathbf{J}^{\mathbf{SH}}} \bar{y}^{SH}_{j,v,t-1,s} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SE}}} \bar{y}^{SE}_{j,v,t-1,s} - \sum\limits_{j \in \mathbf{J}^{\mathbf{SC}}} \bar{y}^{SC}_{j,v,t-1,s}, \quad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t \setminus \mathbf{V}^{\mathbf{N}}_t, s \in \mathbf{S},
\tag{53}
$$

$$\bar{y}_{vts}^P = Y_{vt}^{NB} w, \qquad\qquad t \in \mathbf{T} : t < T^L, v \in \mathbf{V}_t^{\mathbf{N}}, s \in \mathbf{S}, \qquad (54)$$

$$\bar{y}_{vts}^P = \sum_{j \in \mathbf{J}^{\mathbf{NB}}} \bar{y}_{j,v,t-T^L,s}^{NB}, \qquad\qquad t \in \mathbf{T} : t \geq T^L, v \in \mathbf{V}_t^{\mathbf{N}}, s \in \mathbf{S}, \qquad (55)$$

$$\bar{y}_{v0s}^P = Y_v^{IP} w, \qquad\qquad v \in \mathbf{V}_0 \setminus \mathbf{V}_0^{\mathbf{N}}, s \in \mathbf{S}, \qquad (56)$$

$$\bar{y}_{vts}^P = \sum_{j \in \mathbf{J}^{\mathbf{SC}}} \bar{y}_{jvts}^{SC}, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t \setminus \mathbf{V}_{t+1}, s \in \mathbf{S}, \qquad (57)$$

$$\sum_{j \in \mathbf{J}^{\mathbf{SH}}} \bar{y}_{jvts}^{SH} \leq Y_{vts}^{SH} w, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (58)$$

$$\sum_{j \in \mathbf{J}^{\mathbf{SE}}} \bar{y}_{jvts}^{SE} \leq Y_{vts}^{SE} w, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (59)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{j \in \mathbf{J}^{\mathbf{SH}}} \bar{y}_{jvts}^{SH} \leq Y_{ts}^{SH} w, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, s \in \mathbf{S}, \qquad (60)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{j \in \mathbf{J}^{\mathbf{SE}}} \bar{y}_{jvts}^{SE} \leq Y_{ts}^{SE} w, \qquad\qquad t \in \mathbf{T} \setminus \{\bar{T}\}, s \in \mathbf{S}. \qquad (61)$$

$$\bar{l}_{vts}^U - \bar{h}_{vts}^I + \bar{h}_{vts}^O \leq \bar{y}_{vts}^P, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (62)$$

$$\bar{h}_{vts}^I \leq H_{vts}^I w, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (63)$$

$$\bar{h}_{vts}^O \leq H_{vts}^O w, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (64)$$

$$\sum_{v \in \mathbf{V}_t} \bar{h}_{vts}^I \leq H_{ts}^I w, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, s \in \mathbf{S}, \qquad (65)$$

$$\sum_{v \in \mathbf{V}_t} \bar{h}_{vts}^O \leq H_{ts}^O w, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, s \in \mathbf{S}, \qquad (66)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_{kv} \bar{x}_{vlts} + \sum_{k \in \mathbf{K}} \bar{h}_{kits}^S \geq D_{kits} w, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{C}}, k \in \mathbf{K}, s \in \mathbf{S}, \qquad (67)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_{kv} \bar{x}_{vlts} \geq D_{kits} \bar{\delta}_{its}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, k \in \mathbf{K}, s \in \mathbf{S}, \qquad (68)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_v \bar{x}_{vlts} \geq \sum_{k \in \mathbf{K}} (D_{kits} w - \bar{h}_{kits}^S), \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{C}}, s \in \mathbf{S}, \qquad (69)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} Q_v \bar{x}_{vlts} \geq \sum_{k \in \mathbf{K}} D_{kits} \bar{\delta}_{its}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}, \qquad (70)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} \bar{x}_{vlts} \geq F_{it} w, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{C}}, s \in \mathbf{S}, \qquad (71)$$

$$\sum_{v \in \mathbf{V}_t} \sum_{l \in \mathbf{L}_{ivt}} \bar{x}_{vlts} \geq F_{it} \bar{\delta}_{its}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}, \qquad (72)$$

$$\sum_{l \in \mathbf{L}_{vt}} Z_{lv} \bar{x}_{vlts} \leq Z_v (\bar{y}_{vts}^P + \bar{h}_{vt}^I - \bar{h}_{vt}^O - \bar{l}_{vts}^U), \qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \qquad (73)$$

$$\bar{\delta}_{its} \leq \bar{\delta}_{i,t+1,s}, \qquad\qquad t \in \mathbf{T} \setminus \{0, \bar{T}\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}. \qquad (74)$$

Objective function (51a)-(51f) and constraints (52)-(74) provide a reformulation of the objective function and constraints of the RMax model. In addition, the following constraints must be added.

$$\sum_{t\in\mathbf{T}}\sum_{s\in\mathbf{S}} p_s \bar{c}_{ts}^E = \bar{T} + 1, \tag{75}$$

$$w - \bar{\delta}_{its} + \delta_{its} \leq 1, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}, \tag{76}$$

$$\bar{\delta}_{its} - w \leq 0, \qquad\qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}, \tag{77}$$

$$\bar{\delta}_{its} - \delta_{its} \leq 0 \qquad\qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N}_t^{\mathbf{O}}, s \in \mathbf{S}, \tag{78}$$

$$w - \bar{y}_{jvts}^{NB} + y_{jvts}^{NB} \leq 1, \qquad j \in \mathbf{J^{NB}}, t \in \mathbf{T} : t \leq \bar{T} - T^L, v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}, s \in \mathbf{S}, \tag{79}$$

$$\bar{y}_{jvts}^{NB} - w \leq 0, \qquad j \in \mathbf{J^{NB}}, t \in \mathbf{T} : t \leq \bar{T} - T^L, v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}, s \in \mathbf{S}, \tag{80}$$

$$\bar{y}_{jvts}^{NB} - y_{jvts}^{NB} \leq 0 \qquad j \in \mathbf{J^{NB}}, t \in \mathbf{T} : t \leq \bar{T} - T^L, v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}, s \in \mathbf{S}, \tag{81}$$

$$w - \bar{y}_{jvts}^{SH} + y_{jvts}^{SH} \leq 1, \qquad\qquad j \in \mathbf{J^{SH}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{82}$$

$$\bar{y}_{jvts}^{SH} - w \leq 0, \qquad\qquad j \in \mathbf{J^{SH}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{83}$$

$$\bar{y}_{jvts}^{SH} - y_{jvts}^{SH} \leq 0 \qquad\qquad j \in \mathbf{J^{SH}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{84}$$

$$w - \bar{y}_{jvts}^{SE} + y_{jvts}^{SE} \leq 1, \qquad\qquad j \in \mathbf{J^{SE}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{85}$$

$$\bar{y}_{jvts}^{SE} - w \leq 0, \qquad\qquad j \in \mathbf{J^{SE}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{86}$$

$$\bar{y}_{jvts}^{SE} - y_{jvts}^{SE} \leq 0 \qquad\qquad j \in \mathbf{J^{SE}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{87}$$

$$w - \bar{y}_{jvts}^{SC} + y_{jvts}^{SC} \leq 1, \qquad\qquad j \in \mathbf{J^{SC}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{88}$$

$$\bar{y}_{jvts}^{SC} - w \leq 0, \qquad\qquad j \in \mathbf{J^{SC}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{89}$$

$$\bar{y}_{jvts}^{SC} - y_{jvts}^{SC} \leq 0 \qquad\qquad j \in \mathbf{J^{SC}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}. \tag{90}$$

Constraints (75) correspond to the relationship introduced by equation (41), while constraints (76)-(90) define the relationships between the original binary variables and the linearization variables as explained by Glover (1975).

$$y_{jvts}^{NB} \leq y_{j-1,v,t,s}^{NB}, \qquad j \in \mathbf{J^{NB}} \setminus \{1\}, t \in \mathbf{T} : t \leq \bar{T} - T^L, v \in \mathbf{V}_{t+T^L}^{\mathbf{N}}, s \in \mathbf{S}, \tag{91}$$

$$y_{jvts}^{SH} \leq y_{j-1,v,t,s}^{SH}, \qquad\qquad j \in \mathbf{J^{SH}} \setminus \{1\}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{92}$$

$$y_{jvts}^{SE} \leq y_{j-1,v,t,s}^{SE}, \qquad\qquad j \in \mathbf{J^{SE}} \setminus \{1\}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}, \tag{93}$$

$$y_{jvts}^{SC} \leq y_{j-1,v,t,s}^{SC}, \qquad\qquad j \in \mathbf{J^{SC}} \setminus \{1\}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V}_t, s \in \mathbf{S}. \tag{94}$$

Constraints (91)-(94) strengthen the formulation by reducing symmetry.

Finally, the domain of the variables is defined in (95)-(112).

$$y_{jvts}^{NB} \in \{0,1\}, \qquad j \in \mathbf{J^{NB}}, t \in \mathbf{T} : t \le \bar{T} - T^L, v \in \mathbf{V_{t+T^L}^N}, s \in \mathbf{S}, \qquad (95)$$

$$y_{jvts}^{SC} \in \{0,1\}, \qquad j \in \mathbf{J^{SC}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (96)$$

$$y_{jvts}^{SH} \in \{0,1\}, \qquad j \in \mathbf{J^{SH}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (97)$$

$$y_{jvts}^{SE} \in \{0,1\}, \qquad j \in \mathbf{J^{SE}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (98)$$

$$\bar{y}_{jvts}^{NB} \in \mathbf{R^+}, \qquad j \in \mathbf{J^{NB}}, t \in \mathbf{T} : t \le \bar{T} - T^L, v \in \mathbf{V_{t+T^L}^N}, s \in \mathbf{S}, \qquad (99)$$

$$\bar{y}_{jvts}^{SC} \in \mathbf{R^+}, \qquad j \in \mathbf{J^{SC}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (100)$$

$$\bar{y}_{jvts}^{SH} \in \mathbf{R^+}, \qquad j \in \mathbf{J^{SH}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (101)$$

$$\bar{y}_{jvts}^{SE} \in \mathbf{R^+}, \qquad j \in \mathbf{J^{SE}}, t \in \mathbf{T} \setminus \{\bar{T}\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (102)$$

$$\bar{y}_{vts}^{P} \in \mathbf{R^+}, \qquad t \in \mathbf{T}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (103)$$

$$\bar{l}_{vts} \in \mathbf{R^+}, \qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (104)$$

$$\bar{h}_{vts}^{I} \in \mathbf{R^+}, \qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (105)$$

$$\bar{h}_{vts}^{O} \in \mathbf{R^+}, \qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V_t}, s \in \mathbf{S}, \qquad (106)$$

$$\bar{x}_{vlts} \in \mathbf{R^+}, \qquad t \in \mathbf{T} \setminus \{0\}, v \in \mathbf{V_t}, l \in \mathbf{L_{vt}}, s \in \mathbf{S}, \qquad (107)$$

$$\bar{h}_{kits}^{S} \in \mathbf{R^+}, \qquad k \in \mathbf{K}, t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N_t^C}, s \in \mathbf{S}, \qquad (108)$$

$$\bar{c}_{ts}^{E} \in \mathbf{R^+}, \qquad t \in \mathbf{T}, s \in \mathbf{S}, \qquad (109)$$

$$\delta_{its} \in \{0,1\}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N_t^O}, s \in \mathbf{S}, \qquad (110)$$

$$\bar{\delta}_{its} \in \mathbf{R^+}, \qquad t \in \mathbf{T} \setminus \{0\}, i \in \mathbf{N_t^O}, s \in \mathbf{S}, \qquad (111)$$

$$w \in \mathbf{R^+}. \qquad (112)$$

C Ship Types and Trades in the instances

Table 6 and Table 7 report the ship types and trades, respectively, used in the instances described in Section 4.

38

| | | Capacity [RT43] | | | Service | Ships in initial fleet | | |
Ship Type	Age	Car	BB	HH	Speed (knots)	L	M	S
PCC1	26	4975	300	2200	18.5	4	4	3
PCC2	12	6800	300	2500	18.5	6	3	2
PCTC1	9	6800	300	2500	19	8	4	3
LCTC1	4	6000	1500	2000	19	10	5	5
PCTC2	4	5450	900	2200	19	12	7	2
PCTC3	14	6150	200	1800	19.6	7	3	3
RORO1	28	4853	1500	3100	20.5	6	3	2
RORO2	-1	5660	2200	4000	20.8	2	0	1
LCTC2	0	6000	1500	2000	18.5	0	2	0
RORO3	-2	5660	2200	4000	20.8	0	0	0
LCTC3	-1	6000	1500	2000	18.5	0	0	0
RORO4	-3	5660	2200	4000	20.8	0	0	0
LCTC4	-2	6000	1500	2000	18.5	0	0	0
LCTC5	-3	6000	1500	2000	18.5	0	0	0

Table 6: Ship types in the instances. A negative age means that the ship can be delivered from year $t = -Age$. 1 RT43 $\approx$ 9.1 m^3

| | | Demand [Units RT43] | | | | Role in instance | | |
Trade	Length [NM]	Car	HH	BB	Frequency	L	M	S
T1	13 500	435 213	77 047	5 762	48	C	C	C
T2	11 700	103 048	14 201	4 476	0	C	C	C
T3	7 800	41 036	24 420	3 404	0	C	C	C
T4	7 500	19 200	0	0	0	C	C	C
T5	13 000	89 406	21 427	7 425	24	C	C	C
T6	6 500	469 379	67 854	20 075	52	C	C	-
T7	14 500	35 331	66 607	36 845	0	C	C	-
T8	7 800	98 000	29 240	2 689	0	C	O	O
T9	4 900	119 397	44 121	2 167	0	C	O	-
T10	9 000	60 159	7 401	1939	48	O	O	O
T11	8 400	24 818	10 434	3 252	0	O	O	O
T12	6 500	14 0508	53 928	14 115	0	O	-	-
T13	15 021	266 855	55 474	16 776	0	C	-	-
T14	19 200	397 688	123 779	66 198	48	C	-	-

Table 7: Trades in the instances. Each Trade can be both Contractual (C) or Optional (O) in different trades.