# Pricing Car-sharing Services in Multi-Modal Transportation Systems: An Analysis of the cases of Copenhagen and Milan 

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#### Abstract

In this article we study the problem of pricing car-sharing services in multi-modal urban transportation systems. The pricing problem takes into account the competition of alternative mobility services such as public transportation and bicycles and incorporates customer preferences by means of utility functions. The problem is formulated as a linear demand-based discrete optimization problem. A case study based on the cities of Copenhagen and Milan suggests that cycling habits and the efficiency of public transportation services have a significant effect on the viability of car-sharing services.


## 1 Introduction

During the past decade, car-sharing systems have become an attractive means of urban mobility in several cities around the world and dozens of companies have been built to provide such novel mobility services. In car-sharing services, customers share the use of a fleet of cars that is owned, maintained, and managed by a Car-sharing Operator (CSO). The customers are typically able to access shared cars without interacting directly with the CSO as reservations, pick ups, and returns are often self-serviced through the internet. Car-sharing services can be divided into two categories, namely free-floating systems and station-based systems. Free-floating systems enable users to pick up and return shared cars at any parking spot within a specified business area. In station-based systems, cars are assigned to dedicated stations and users must pick up and return cars at the specified stations. In this case we distinguish two-way systems, requiring the user to return the car at the pick up station, and one-way systems, allowing the user to return the car at a different station. Users generally pay based on their use of the car in addition to a possible subscription fee, while all vehicle costs are born by the CSO (e.g., fuel, insurance and maintenance).

CSOs face novel challenges at different planning levels which have attracted the interest of the scientific community in recent years. At the strategic level the CSO must decide the fleet size and business area [115], the trip booking scheme [714] and, in station-based systems, the location, number and capacity

[^0]of stations [6|18|3|19]. At the operating level, CSOs face planning problems such as the repositioning of vehicles $15|10| 16|22| 6|29| 13|14| 23 \mid 4$, maintenance $16 \mid 24$, charging and refueling [5/24|17|19].

In this paper we focus on the problem of pricing car-sharing services. Particularly, we look at car-sharing services within the context of multi-modal transportation systems. Classical urban transportation means such as bus, subway, and bicycles, can in fact be seen as competitors of car-sharing services in the market of urban transportation services. Therefore, CSOs need to take into account the alternative transportation means within a city, as well as customer preferences, when deciding about pricing schemes. The preferences of customers are often formalized using specific models such as logit models. However, the resulting integrated models are typically computationally difficult due to the non-linear interaction between the decision variables. In addition, convexification and linearization of such models (see, e.g., [130] might not help to solve real-life intances (see [26]). Therefore, we propose a linear demand-based discrete optimization model in the spirit of [2]. The model explicitly takes into account that customers demand for transportation depends on the price set by the CSO as well as on the characteristics and price of the alternative transportation services. Customers preferences are included in the optimization model by means of a utility function which can be adapted to the specific market. When the utility function is linear in the price, the optimization model can be formulated as a MILP, thus avoiding the non-linearity typically generated by classical choice models.

The contribution of this paper is twofold. First, we provide a novel optimization model for pricing car-sharing services in multi-modal transportation systems which explicitly takes into account customers preferences and the competition of alternative transportation means. Second, we offer an analysis of car-sharing services in Copenhagen and Milan which investigates the influence of different characteristics of public transportation services. Similarly, 12 addressed the effects of relocation in a car-sharing service in Hamburg, [25] provided an empirical analysis of car-sharing usage in Munich and Berlin, and [20] studied the elements driving satisfaction for bike-sharing users in Milan.

In Section 2 we describe the pricing problem and in Section 3 we introduce the corresponding mathematical model. In Section 4 we use the model to study the cases of Copenhagen and Milan, while in Section 5 we draw final conclusions.

## 2 Problem Description

We consider a CSO operating in a city which offers a number of (private or public) transportation services (e.g., buses, metro, cycling lines). The CSO must determine the price of car-sharing rides. CSOs typically charge a per-minute fee plus a constant drop-off fee which depends on the zone of the city where the car is returned. For instance Car2Go (www.car2go.com, a CSO operating in several cities around the globe) divides Milan in zone A (comprising the city center and its surroundings) and B (comprising the outskirts of the city) and charges $€ 4.90$
when returning the car in zone $B$ (no extra charge for zone $A)^{17}$ Consistently with common practice, we assume a pricing scheme made of a per-minute fee and a drop-off fee. However, we generalize such pricing scheme by assuming the dropoff fee depends on the customer's origin and destination (O-D) pair, while the per-minute fee is common to all O-D pairs. Such pricing scheme allows the CSO to consider the city's specific transportation means at a higher level of granularity and price car-sharing rides according to the specific O-D pair, thus taking into account the competition on individual routes. In addition, it provides the CSO an instrument to offer customers incentives for moving the cars in accordance with some ideal distribution plan, and thus reducing the need for staff-based repositioning of cars. However, this requires that, upon booking, the CSO is able to inform their users about the drop-off fees based on their current location and all possible destinations.

Given an O-D pair, customers can choose between a number of transportation services. The set of available transportation services depends on the specific OD pair. The demand for car-sharing rides between an O-D pair depends on the customers personal preferences and on the characteristics of the available transportation services, such as price, travel time, and waiting time. Specifically, a customer's choice depends on the utility obtained by choosing a service, and each customer chooses the service that gives them the highest utility.

Therefore, given an O-D pair within the city, the available transportation services, their prices and characteristics, the set of customer types characterized by their utility functions, the CSO's problem of pricing car-sharing services consists of deciding i) whether to offer car-sharing services between the given O-D pair and ii) the O-D pair specific drop-off fee in order to maximize its profit.

## 3 Mathematical Model

We formulate the problem usign the demand-based discrete optimization framework proposed by [2] which entails modeling customers response to pricing decisions by means of a utility function. We begin by clarifying the necessary modeling assumptions in Section 3.1 and, following, we introduce the notation and the mathematical model in Section 3.2

### 3.1 Modeling Assumption

We assume that the market for urban transportation between an O-D pair within the city consists of a finite number of customers or, alternatively, of a finite number of groups of customers with homogeneous behavior. We also assume that, for the given O-D pair, the set of transportation services, their prices and a list of their features (e.g., travel time and waiting time) is known to the CSO and to the customers, that price and characteristics are identical for all customers, and that all transportation services are available to all customers. However, the

[^1]CSO might decide not to offer car-sharing services between a given O-D pair if unprofitable. Furthermore, we assume that the market is closed, meaning that every customer must choose exactly one transportation service.

We assume that each (group of) customer(s) is characterized by a utility function. The utility function is a real-valued function of the characteristics of the transportation services. Each customers values each characteristic differently according to their utility function. We assume that each customer chooses the available service which gives them the highest utility. In practice, the utility function is not fully known to the CSO. Therefore, we assume that the actual utility for a customer is a random variable for the CSO. An example of utility function will be given in Section 4.1 .

We assume that the CSO offers a pricing scheme consisting of a per-minute fee common to all O-D pairs, plus a drop-off fee which is O-D specific and must be decided by the CSO. We assume that the drop-off fee is known by the customers upon reserving a shared car. Finally, for the sake of simplicity, we assume that users drive directly from the origin to the destination. This assumption can be easily relaxed by assuming user-specific paths trough the city.

### 3.2 Notation and Model

In this section we first introduce the notation and then the optimization model.

| Sets |  |
| :---: | :---: |
| $\mathcal{C}$ | the set of customers or groups of customers |
| $\mathcal{S}$ | the set of all transportation services |
| $\mathcal{S}^{C S} \subseteq \mathcal{S}$ | the set of transportation services offered by the CSO, such as different models of shared cars |
| $\mathcal{R}$ | the set of utility scenarios |
| $\mathcal{L}_{s}$ | the set of possible drop-off fee levels for service $s \in \mathcal{S}^{C S}$ |
| Parameters |  |
| $P_{s}^{M}$ | the price-per-minute of car-sharing service $s \in \mathcal{S}^{C S}$ |
| $P_{s l}^{D}$ | the drop-off fee at level $l \in \mathcal{L}_{s}$ for car-sharing service $s \in \mathcal{S}^{C S}$ |
| $P_{s}$ | the price of transportation service $s \in \mathcal{S} \backslash \mathcal{S}^{C S}$ |
| $T_{s}^{C S}$ | the travel time between the given O-D using car-sharing service $s \in \mathcal{S}^{C S}$ |
| $C_{\text {sc }}$ | the cost of offering car-sharing service $s \in \mathcal{S}^{C S}$ to customer $c \in \mathcal{C}$ on the given O-D pair |
| $\epsilon_{s c r}$ | realization of the random utility error for service $s \in \mathcal{S}$ and customer $c \in \mathcal{C}$ under scenario $r \in \mathcal{R}$ |
| $M_{c r}$ | upper bound on the difference in utility between two services for customer $c \in \mathcal{C}$ in scenario $r \in \mathcal{R}$ |
| $\pi_{s}^{1}, \ldots, \pi_{s}^{N}$ | a list of $N$ attributes for transportation service $s \in \mathcal{S}$ |
| $f_{c}: \mathbb{R}^{N+1} \rightarrow \mathbb{R}$ the utility function for customer $c \in \mathcal{C}$ |  |
| Variables |  |
| $p_{s}$ | the price for service $s \in \mathcal{S}$ |


| $u_{s c r}$ | the utility obtained by customer $c \in \mathcal{C}$ for service $s \in \mathcal{S}$ under scenario $r \in \mathcal{R}$ |
| :---: | :---: |
| $y_{s c}$ | a binary variable taking value 1 if service $s \in \mathcal{S}$ is offered to customer $c \in \mathcal{C}, 0$ otherwise |
| $y_{s c r}$ | a binary variable taking value 1 if service $s \in \mathcal{S}$ is offered to customer $c \in \mathcal{C}$ under scenario $r \in \mathcal{R}, 0$ otherwise |
| $w_{s c r}$ | a binary variable taking value 1 , if service $s \in \mathcal{S}$ is chosen by customer $c \in \mathcal{C}$ under scenario $r \in \mathcal{R}, 0$ otherwise |
| $\lambda_{s l}$ | a binary variable taking value 1 , if price level $l \in \mathcal{L}_{s}$ is chosen for service $s \in \mathcal{S}^{C S}, 0$ otherwise |
| $\mu_{s z c r}$ | a binary variable taking value 1 if customer $c \in \mathcal{C}$ obtains a higher utility by choosing service $s \in \mathcal{S}$ over service $z \in \mathcal{S}$ under scenario $r \in \mathcal{R}, 0$ otherwise |
| $\eta_{s z c r}$ | a binary variable taking value 1 if both service $s \in \mathcal{S}$ and $z \in \mathcal{S}$ are available to customer $c \in \mathcal{C}$ under scenario $r \in \mathcal{R}, 0$ otherwise |
| $\alpha_{s c r l}$ | a binary variable taking value 1 if service $s \in \mathcal{S}^{C S}$ is chosen by customer $c \in \mathcal{C}$ under scenario $r \in \mathcal{R}$ at price level $l \in \mathcal{L}_{s}, 0$ otherwise |

The problem of pricing car-sharing services between a given O-D pain can thus be stated as follows.

$$
\begin{array}{ll}
\max & \sum_{s \in \mathcal{S}^{C S}}\left(P_{s}^{M} T_{s}^{C S}+\frac{1}{|\mathcal{R}|} \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}} \sum_{l \in \mathcal{L}_{s}} P_{s l}^{D} \alpha_{s c r l}\right)-\sum_{s \in \mathcal{S}} \sum_{c \in \mathcal{C}} C_{s c} y_{s c} \\
\text { s.t. } & u_{s c r}=f_{c}\left(p_{s}, \pi_{s}^{1}, \ldots, \pi_{s}^{N}\right)+\epsilon_{s c r} \quad c \in \mathcal{C}, s \in \mathcal{S}, r \in \mathcal{R}, \\
& p_{s}=P_{s}^{M} T_{s}^{C S}+\sum_{l \in \mathcal{L}_{s}} P_{s l}^{D} \lambda_{s l} \\
& s \in \mathcal{S}^{C S}, \\
& \\
p_{s}=P_{s} & s \in \mathcal{S} \backslash \mathcal{S}^{C S},  \tag{1e}\\
M_{c r} \eta_{s z c r}-2 M_{c r} \leq u_{s c r}-u_{z c r}-M_{c r} \mu_{s z n r}
\end{array}
$$

$$
c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}
$$

$$
\begin{equation*}
u_{s c r}-u_{z c r}-M_{c r} \mu_{s z c r} \leq\left(1-\eta_{s z c r}\right) M_{c r} \tag{1f}
\end{equation*}
$$

$$
c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}
$$

$$
\begin{array}{ll}
\mu_{s z c r}+\mu_{s z c r} \leq 1 & c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}, \\
y_{s c r}+y_{z c r} \leq 1+\eta_{s z c r} & c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}, \\
\eta_{s z c r} \leq y_{s c r} & c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}, \\
\eta_{s z c r} \leq y_{z c r} & c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}, \\
\mu_{s z c r} \leq y_{s c r} & c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}, \\
w_{s c r} \leq \mu_{s z c r} & c \in \mathcal{C}, s \neq z \in \mathcal{S}, r \in \mathcal{R}, \tag{11}
\end{array}
$$

$$
\begin{array}{lr}
\sum_{s \in \mathcal{S}} w_{s c r}=1 & c \in \mathcal{C}, r \in \mathcal{R}, \\
\lambda_{s l}+w_{s c r} \leq 1+\alpha_{s c r l} & c \in \mathcal{C}, s \in \mathcal{S}^{C S}, r \in \mathcal{R}, l \in \mathcal{L}_{s}, \\
\alpha_{s c r l} \leq \lambda_{s l} & c \in \mathcal{C}, s \in \mathcal{S}^{C S}, r \in \mathcal{R}, l \in \mathcal{L}_{s}, \\
\alpha_{s c r l} \leq w_{s c r} & c \in \mathcal{C}, s \in \mathcal{S}^{C S}, r \in \mathcal{R}, l \in \mathcal{L}_{s}, \\
\sum_{l \in \mathcal{L}_{s}} \lambda_{s l}=1 & s \in \mathcal{S}^{C S}, \\
y_{s c r} \leq y_{s c} & c \in \mathcal{C}, s \in \mathcal{S}^{C S}, r \in \mathcal{R}, \\
y_{s c}=1 & c \in \mathcal{C}, s \in \mathcal{S} \backslash \mathcal{S}^{C S}, \\
y_{s c r}=1 & c \in \mathcal{C}, s \in \mathcal{S} \backslash \mathcal{S}^{C S}, r \in \mathcal{R}, \\
p_{s} \geq 0 & s \in \mathcal{S}, \\
y_{s c} \in\{0,1\} & c \in \mathcal{C}, s \in \mathcal{S}, \\
y_{s c r}, w_{s c r} \in\{0,1\} & c \in \mathcal{C}, s \in \mathcal{S}, r \in \mathcal{R}, \\
\lambda_{s l} \in\{0,1\} & s \in \mathcal{S}, \\
\mu_{s z c r}, \eta_{s z c r} \in\{0,1\} & , l \in \mathcal{L}, \\
\alpha_{s c r l} \in\{0,1\} & c \in \mathcal{C}, s \in \mathcal{S}^{C S}, r \in \mathcal{R}, l \in \mathcal{L} s .
\end{array}
$$

Objective function 1a) represents the expected profit generated on the given O-D pair. Constraints 1 Bb define the utility as the sum of a customer-specific utility dependent on the attributes of the transportation systems (the part of the utility the CSO can explain) and a random term $\epsilon_{s c r}$ which plays the twofold role of describing the component of the utility that the CSO cannot explain as well as possible irrational customer choices. When $f_{c}\left(p_{s}, \pi_{s}^{1}, \ldots, \pi_{s}^{N}\right)$ is a linear in $p_{s}$ model (1) is a MILP. However, it is not required that $f_{c}(\cdot)$ is linear in the remaining attributes $\pi_{s}^{1}, \ldots, \pi_{s}^{N}$. In Section 4.1 we introduce a specific utility function based on the available literature. Constraints 1 c ) and 1 dd set the price for the transportation services offered by the CSO (the sum of per-minute and drop-off fee) and by other parties, respectively. Constraints 1e) and (1f) ensure that, among two services a customer always chooses the one with the highest utility. Constraints 1g ensure that, given services $s$ and $z$, either $s$ has a higher utility than $z$ or viceversa. Constraints 1 h ) ensure that $\eta_{s z c r}$ takes value 1 if both service $s$ and $z$ are offered to customer $c$ under scenario $r$. Consistently, constraints (1i) and (1j) ensure that variable $\eta_{s z c r}$ takes value 0 if either service $s$ or $z$ are not offered to customer $c$ under scenario $r$. Constraints 1 k state that service $s$ cannot be preferred to service $z$ by customer $c$ under scenario $r$ if the service is not offered to the customer. Constraints (11) state that customer $c$ can choose service $s$ only if its utility is the highest in scenario $r$. Constraints (1m) ensure that each customer chooses exactly one service. Constraints (1n) - (1p) are required in order to obtain a linear objective function. Constraints (1n) ensure that $\alpha_{s c r l}$ takes value 1 if price level $l$ has been chosen for service $s$ and customer $c$ has chosen service $s$ under scenario $r$. Constraints 10 and (1p) ensure that $\alpha_{s c r l}$ takes value 0 if price level $l$ has not been chosen and if
customer $c$ has not chosen service $s$, respectively. Constraints (1q) ensure that only one price level is selected. Constraints (1r) ensure that if a service is not offered to customer $c$ it is not offered in any of the scenarios. Constraints 1s) and (1t) ensure that the transportation services other than car-sharing are always available to all users. Finally, constraints (1u) - 1 z ) define the domain for the decision variables.

## 4 The Cases of Copenhagen and Milan

In this section we use model (1) to investigate the profitability of car-sharing services in the cities of Copenhagen, Denmark, and Milan, Italy. Particularly, the scope of the computational study is to analyze the price a CSO is able to set between different zones of the cities, and the corresponding market response. Model (1) has been implemented in GAMS 24.4.6 and solved using CPLEX on a machine with 4 GB RAM and a 2.3 GHz CPU .

Car-sharing services have been adopted in both cities. To our knowledge only one free-floating car-sharing service is operating in Copenhagen as of January 2018, while at least four can be counted in Milan. In both cities there exists a public transportation provider offering services such as buses, metro lines, and surface/underground trains. Cycling trails reach a higher level of capillarity in Copenhagen, where bicycles are a common transportation option. According to [9] nine out of ten Danes own a bicycle and in 2016 the number of bicycles crossing the city center of Copenhagen exceeded the number of cars. On the contrary, cycling is not as popular in Milan to the extent that the municipality is seeking economic incentives to improve cycling mobility [27]. Therefore, for the city of Copenhagen we consider three transportation services, namely carsharing, public transportation, and bicycles while for Milan we consider carsharing and public transportation. In both cities, public transportation between a given O-D pair may include commuting and, for the sake of simplicity, we assume bicycles cannot be taken on board public transportation.

In Section 4.1 we describe the utility function used in the computational study and the groups of customers considered. In Section 4.2 we describe the attributes of the transportation services. Finally, in Section 4.3 we discuss the results obtained.

### 4.1 Utility Function

We use the utility function provided by [21 with minor adjustments to our specific case. The function is linear in the price $p_{s}$ rendering model (1) is a MILP. For each $s \in \mathcal{S}$ and $c \in \mathcal{C}$ the utility can be stated as 22.

$$
\begin{array}{r}
f_{c}\left(p_{s}, T_{s}^{C S}, T_{s}^{P T}, T_{s}^{B}, T_{s}^{W}, T_{s}^{\text {Wait }}\right)=\beta_{c}^{P} p_{s}+\beta_{c}^{C S} T_{s}^{C S}+\beta_{c}^{P T} T_{s}^{P T} \\
+\tau\left(T_{s}^{B}\right) \beta_{c}^{B} T_{s}^{B}+\tau\left(T_{s}^{W}\right) \beta_{c}^{W} T_{s}^{W}+\beta_{c}^{\text {Wait }} T_{s}^{\text {Wait }} \tag{2}
\end{array}
$$

Here, $T_{s}^{C S}$ represents the time spent riding a shared car, $T_{s}^{P T}$ the total time spent in public transportation, $T_{s}^{B}$ the time spent riding a bicycle, $T_{s}^{W}$ the walking time
which includes the walking time to the nearest transportation service (such as a shared car or bus stop), between public transportation means, and to the final destination and, finally, $T_{s}^{\text {Wait }}$ the total waiting time. The $\beta$ coefficients of $(2)$ are quantified following the procedure illustrated by [21] (after converting in Euro the values provided in Italian Liras when necessary). Two customer segments are introduced, namely lower-middle class (LMC) and upper-middle class (UMC), thus $\mathcal{C}=\{L M C, U M C\}$. We obtain $\beta_{c}^{P}=-188.33$ and $\beta_{c}^{P}=-70.63$, for $c=L M C$ and $U M C$, respectively. Furthermore, we set $\beta_{c}^{C S}=-1, \beta_{c}^{P T}=-2$, $\beta_{c}^{B}=-2.5, \beta_{c}^{W}=-3$ and $\beta_{c}^{W \text { ait }}=-6$ for all $c \in \mathcal{C}$. The function $\tau: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $\tau(t)=\left\lceil\frac{t}{10}\right\rceil$ and allows us to model the utility of cycling and walking as a piece-wise linear function representing the fact that the utility of walking and cycling decreases faster as the walking and cycling time increases.

Finally, uncertainty in the preferences of customers is considered by creating $|\mathcal{R}|=100$ utility scenarios. Each scenario consists of a realization of the error term $\epsilon_{s c r}=\xi_{s c r} f_{c}\left(p_{s}, T_{s}^{C S}, T_{s}^{P T}, T_{s}^{B}, T_{s}^{W}, T_{s}^{\text {Wait }}\right)$, where $\xi_{s c r}$ is an i.i.d $\mathcal{N}(0,0.1)$ sample. This corresponds to assuming a normally distributed error with a $10 \%$ standard deviation.

### 4.2 Characteristics of the Cities

We consider a base case which includes car-sharing, public transportation, and bicycle for Copenhagen and car-sharing and public transportation for Milan. However, the influence of cycling habits in both cities is investigated in Section 4.4. Copenhagen and Milan have been divided into eight and ten evently spread zones, respectively. For each zone a central point acts as origin/destination. For each city, O-D pair, and transportation service $s \in \mathcal{S}$, the values of the attributes $p_{s}, T_{s}^{C S}, T_{s}^{P T}, T_{s}^{B}, T_{s}^{W}, T_{s}^{W a i t}$ are calculated based on the actual transportation services and distances. For each transportation service, we assume customers always choose the fastest option (e.g., driving route or public transportation connection). The fastest driving and cycling routes are found through Google Maps. The fastest public transportation connections are found through Rejseplanen (www.rejseplanen.dk) for Copenhagen and Google Maps and ATM (www.atm.it) for Milan. We assume a cycling speed of 16 kilometers/hour, which includes stops at traffic lights and a walking speed of 5 kilometers/hour. Furthermore, we assume shared cars are always available within 500 meters from the origin. The impact of a reduced distance from shared cars is investigated in Section 4.4. All the time-related attributes for each O-D pair and transportation services are provided in Appendix A.

The price for bicycle rides is always zero, while the prices of public transportation services are taken from the local providers and are $1.60 €$ for all O-D pairs in Copenhager ${ }^{2}$ and $1.5 €$ for each O-D pair in Milan. Finally, the price of car-sharing services is set according to current market prices. Particularly, we register that in Milan the per-minute fee offered as of January 2018 varies

[^2]between 0.24 and $0.29 € / \mathrm{min}$ between the different CSO. We adopt a lower perminute fee, namely $0.20 € / \mathrm{min}$, in order to assess the opportunity of including an O-D specific drop-off fee. We consider four possible drop-off fees, namely $0,1,2$ and $3 €$. In Section 4.4 the influence of different per-minute fees is investigated. Finally, for the sake of simplicity, the cost of car-sharing services is ignored, i.e., $C_{s c}=0$ for all $s \in \mathcal{S}^{C S}$ and $c \in \mathcal{C}$ so that we consider the maximization of the revenue, and we assume a trip from $O$ to $D$ has the same characteristics as a trip from $D$ to $O$.

### 4.3 Results for the Base Case

Table 2 and Table 3 report, for each O-D pair in Copenhagen and Milan, respectively, the CSO's expected revenue (assuming one customer for each segment), the chosen drop-off fee, and the distribution of customers among transportation services (alternatively the probability that the customer chooses a transportation service). Based on the results in Table 2 and Table 3, car-sharing appears much more competitive in Milan than in Copenhagen. In Copenhagen, the CSO makes a positive revenue only on one O-D pair, while in Milan the CSO makes a positive revenue on almost all the O-D pairs. In Copenhagen, the great majority of the customers is attracted by the possibility of cycling (inexpensive and relatively easy due to the short distances). It can be noticed that the O-D pair Østerbro-Ørestad, the only O-D pair for which the CSO makes a profit in Copenhagen, is also the only one with a cycling distance longer than 30 minutes. On the other hand, Table 3 shows that in Milan, despite public transportation services are a serious competitor (especially for the LMC customers), car-sharing services can attract a fair percentage of customers. However, the results show that the CSO does not have enough market power to charge a drop-off fee. The competitiveness of car-sharing services is highly price-sensitive, and the viability of car-sharing services depends on the cost or running the service.
Table 2: Results for Copenhagen. The expected revenue assumes one customer for each customer group. $\% \mathrm{CS}, \% \mathrm{PT}$ and $\% \mathrm{~B}$ indicate the percentage of customers choosing car-sharing, public transportation and bicycle, respectively.

| Origin | Destination | Expected <br> Revenue [ $€$ ] | $P_{i l}^{D}[€] .$ | \% CS |  | \% PT |  | \% B |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | LMC | UMC | LMC | UMC | LMC | UMC |
| $\emptyset$ sterbro | København K | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| $\emptyset$ sterbro | Nørrebro | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Østerbro | Fredriksberg C | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Østerbro | Frederiksberg | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| $\emptyset$ sterbro | Vesterbro | 0 | 0 | 0 | 0 | 0 | 7 | 100 | 93 |
| $\emptyset$ sterbro | $\emptyset$ restad | 0.352 | 0 | 0 | 8 | 0 | 72 | 100 | 20 |
| Østerbro | $\emptyset$ st Amager | 0 | 0 | 0 | 0 | 0 | 34 | 100 | 66 |
| København K | Nørrebro | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| København K | Fredriksberg C | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| København K | Frederiksberg | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| København K | Vesterbro | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| København K | $\emptyset$ restad | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| København K | $\varnothing$ st Amager | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Nørrebro | Fredriksberg C | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Nørrebro | Frederiksberg | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Nørrebro | Vesterbro | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Nørrebro | $\emptyset$ restad | 0 | 0 | 0 | 0 | 0 | 3 | 100 | 97 |
| Nørrebro | $\varnothing_{\text {st }}$ Amager | 0 | 0 | 0 | 0 | 0 | 28 | 100 | 72 |
| Fredriksberg C | Frederiksberg | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Fredriksberg C | Vesterbro | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |


| Fredriksberg C | Ørestad | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| Fredriksberg C | Øst Amager | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Frederiksberg | Vesterbro | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Frederiksberg | $\varnothing$ restad | 0 | 0 | 0 | 0 | 0 | 3 | 100 | 97 |
| Frederiksberg | Øst Amager | 0 | 0 | 0 | 0 | 0 | 25 | 100 | 75 |
| Vesterbro | $\varnothing$ restad | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |
| Vesterbro | $\varnothing$ st Amager | 0 | 0 | 0 | 0 | 0 | 1 | 100 | 99 |
| $\varnothing$ restad | $\varnothing$ st Amager | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 100 |

The lower competitiveness of car-sharing services in Copenhagen is consistent, for example, with a statement released by Car2Go upon closing their service in Copenhagen (reported by [8] and [28]): "Car2Go has not reached the critical mass in demand necessary to establish a successful, viable and robust business in Denmark". Our analysis suggests that cycling habits might be one of the main reasons behind the different successes of car-sharing services in Copenhagen and Milan. This is further investigated in Section 4.4. However, the necessary simplification made in our analysis might also influence the results. Particularly, we categorized customers based only on their price sensitivity while further discrimination by e.g., age and health conditions, might provide additional insights.

Table 3: Results for Milan. The expected revenue assumes one customer for each customer group. \%CS and \%PT indicate the percentage of customers choosing car-sharing and public transportation, respectively.

| Origin | Destination | $\begin{gathered} \text { Expected } \\ \text { Revenue }[€] \end{gathered}$ | $P_{i l}^{D}[€]$ | \% CS |  | \% PT |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | LMC | UMC | LMC | UMC |
| Portobello | Derganino | 2.882 | 0 | 34 | 97 | 66 | 3 |
| Portobello | China Town | 3.060 | 0 | 70 | 100 | 30 | 0 |
| Portobello | Sempione | 2.880 | 0 | 63 | 97 | 37 | 3 |
| Portobello | Washinghton | 4.158 | 0 | 89 | 100 | 11 | 0 |
| Portobello | Carrobbio | 0.432 | 0 | 0 | 12 | 100 | 88 |
| Portobello | Ticinese | 0.378 | 0 | 0 | 9 | 100 | 91 |
| Portobello | Guastalla | 0.054 | 0 | 0 | 1 | 100 | 99 |
| Portobello | QDM | 0.294 | 0 | 0 | 7 | 100 | 93 |
| Portobello | Central Station | 0.324 | 0 | 0 | 9 | 100 | 91 |
| Derganino | China Town | 2.912 | 0 | 82 | 100 | 18 | 0 |
| Derganino | Sempione | 2.496 | 0 | 11 | 85 | 89 | 15 |
| Derganino | Washinghton | 2.210 | 0 | 0 | 65 | 100 | 35 |
| Derganino | Carrobbio | 0 | 0 | 0 | 0 | 100 | 100 |
| Derganino | Ticinese | 0 | 0 | 0 | 0 | 100 | 100 |
| Derganino | Guastalla | 0.044 | 0 | 0 | 1 | 100 | 99 |
| Derganino | QDM | 0 | 0 | 0 | 0 | 100 | 100 |
| Derganino | Central Station | 0.858 | 0 | 0 | 33 | 100 | 67 |
| China Town | Sempione | 3.008 | 0 | 88 | 100 | 12 | 0 |
| China Town | Washinghton | 2.990 | 0 | 18 | 97 | 82 | 3 |
| China Town | Carrobbio | 0.224 | 0 | 0 | 7 | 100 | 93 |
| China Town | Ticinese | 0.360 | 0 | 0 | 9 | 100 | 91 |
| China Town | Guastalla | 0.528 | 0 | 0 | 12 | 100 | 88 |
| China Town | QDM | 0.324 | 0 | 0 | 9 | 100 | 91 |
| China Town | Central Station | 0.810 | 0 | 0 | 27 | 100 | 73 |
| Sempione | Washinghton | 2.744 | 0 | 96 | 100 | 4 | 0 |
| Sempione | Carrobbio | 3.132 | 0 | 74 | 100 | 26 | 0 |
| Sempione | Ticinese | 2.928 | 0 | 25 | 97 | 75 | 3 |
| Sempione | Guastalla | 1.938 | 0 | 0 | 57 | 100 | 43 |
| Sempione | QDM | 0.540 | 0 | 0 | 18 | 100 | 82 |
| Sempione | Central Station | 0.038 | 0 | 0 | 1 | 100 | 99 |
| Washinghton | Carrobbio | 3.220 | 0 | 61 | 100 | 39 | 0 |
| Washinghton | Ticinese | 3.072 | 0 | 31 | 97 | 69 | 3 |
| Washinghton | Guastalla | 2.496 | 0 | 2 | 76 | 98 | 24 |
| Washinghton | QDM | 0.324 | 0 | 0 | 9 | 100 | 91 |
| Washinghton | Central Station | 0.046 | 0 | 0 | 1 | 100 | 99 |
| Carrobbio | Ticinese | 2.416 | 0 | 65 | 86 | 35 | 14 |
| Carrobbio | Guastalla | 1.876 | 0 | 2 | 65 | 98 | 35 |
| Carrobbio | QDM | 0.030 | 0 | 0 | 1 | 100 | 99 |
| Carrobbio | Central Station | 0 | 0 | 0 | 0 | 100 | 100 |
| Ticinese | Guastalla | 3.136 | 0 | 96 | 100 | 4 | 0 |
| Ticinese | QDM | 1.638 | 0 | 4 | 59 | 96 | 41 |
| Ticinese | Central Station | 0.304 | 0 | 0 | 8 | 100 | 92 |
| Guastalla | QDM | 1.680 | 0 | 6 | 64 | 94 | 36 |
| Guastalla | Central Station | 1.020 | 0 | 0 | 34 | 100 | 66 |
| QDM | Central Station | 0.676 | 0 | 0 | 26 | 100 | 74 |

### 4.4 Factors Influencing Car-Sharing Services

We investigate the influence of cycling habits by assessing the profitability of car-sharing services in Copenhagen after excluding the possibility of cycling, and in Milan after including the possibility of cycling. The results show that the CSO makes a positive revenue in 24 out of 28 O-D pairs in Copenhagen when the possibility of cycling is excluded. For these O-D pairs, a fair amount of (particularly UMC) customers chooses car-sharing services and, in a number of O-D pairs, car-sharing services are selected more than public transportation, especially when public transportation connections require commuting and waiting. However, also in this case the CSO does not have market power to charge a drop-off fee. In the city of Milan, a dramatic migration of customers from carsharing and public transportation towards bicycles can be observed. For each O-D pair considered, almost all customers choose to move by bicycle. These results are certainly influenced by the simplifications in the utility function which does not include elements such as the purpose of the trip, weather conditions and carry-on items. However, the results clearly illustrate a trend towards bicycles should they become an actually viable transportation system. Thus, it emerges that cycling represents a though competitor to take into account when setting up and pricing car-sharing services. Furthermore, it emerges that CSOs can define better pricing by looking at the configuration of the public transportation systems and particularly at O-D pairs with inefficient connections due to, e.g., long waiting time.

In the cases discussed so far the per-minute fee was $0.20 € / \mathrm{min}$, a tariff lower than current market prices in order to assess the possibility to set an O-Dspecific drop-off fee. We assess three alternative per-minute fees, namely 0.30 (just above market prices), 0.25 (about average market price), and $0.15 € / \mathrm{min}$ (significantly lower than market prices). As intuition suggests, the results show that customers, of both customers classes, shift towards car-sharing services as the per-minute fee decreases. For the case of Milan, the total expected revenue decreases by $67.62 \%$ (with respect to the base case discussed in Section 4.3) with a per-minute fee of $0.30 €$, and by $39.43 \%$ with a per-minute fee of 0.25 $€$, but increases by $53.11 \%$ with a per-minute fee of $0.15 € d u e$ to the significant increase in car-sharing demand. These results show that the per-minute fee is a crucial parameter to influence the penetration of car-sharing services in a city. However, the possibility to impose a drop-off fee remains limited even with a very low per-minute fee.

CSOs determine the proximity of shared cars to users by adjusting the size and distribution of the fleet. In order to assess how the proximity to a shared car influences customers choices and pricing decisions, we consider the base case of Milan and we assume a (possibly unrealistic) zero distance to shared cars. Similar scenarios may be obtained for example with a very large fleet of cars. The results illustrate that, with respect to Table 3 (where the distance to the nearest car is 500 meters), the percentage of customers choosing car-sharing services generally increases and, consequently, the total expected revenue. However, car-sharing does not attract customers on the four O-D pairs where it was never selected
in the base case, illustrating that when public transportation connection are particularly advantageous, car-sharing has little room for gaining market shares. Also in this case the drop-off fee is set to zero on all O-D pairs. Thus, while increased proximity of shared cars can attract more customers and increase the revenue (by $19.63 \%$ in our case), it does not provide CSOs the possibility to replace good public transportation connections, nor enough market power to set a drop-off fee.

Finally, in order to study the effect of public transportation frequency we study the base case of Milan with waiting times increased by $50 \%$. The results show that some LMC customers choose car-sharing in 22 O-D pairs against the 19 of the basic case. For 11 out of 45 O-D pairs all UMC customers choose car-sharing services, against the 8 of the basic case. As a consequence, the total expected revenue increases by $21.41 \%$. Cities with inefficient public transportation services appear therefore a better environment for car-sharing services. This also illustrates the potential of defining pricing strategies which vary with the frequency and configuration of public transportation services.

## 5 Conclusions

This paper presented novel optimization model for pricing car-sharing services taking into account alternative transportation means as well as customers preferences via a utility function. When the utility function is linear in the price of car-sharing services the model can be formulated as a MILP. The model is amenable to further characterizations and enhancements, and to be integrated into broader analytic tools for car-sharing services.

The model is used to illustrate the viability of car-sharing services in Copenhagen and Milan. The study shows that cycling habits have a crucial impact on the market response to car-sharing. Furthermore, it emerges that companies have little margins to increase prices, mainly due to the competition of classical transportation services. However, a richer characterizations of customers preferences might illustrate market power which was not captured by our study. Furthermore, our results show that inefficiency in public transportation services such as long waiting times (due to e.g., low frequency) can be exploited by CSOs to gain market shares.

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## A Attributes of the Origin-Destination pairs considered

Table 4: Time-related attributes of car-sharing (CS), public transportation (PT), and bicycle (B) for the O-D pairs of interest in Copenhagen and Milan.

| City | Origin | Destination | Service | Attributes (min) |  |  |  |  | City | Origin | Destination | Service | Attributes (min) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $T_{s}^{C S}$ | $T_{s}^{P T}$ | $T_{s}^{W}$ | $T_{s}^{B}$ | $T_{s}^{W a i t}$ |  |  |  |  | $T_{s}^{C S}$ | $T_{s}^{P T}$ | $T_{s}^{W}$ | $T_{s}^{B}$ | $T_{s}^{W a i t}$ |
| Copenhagen |  |  |  |  |  |  |  |  |  |  | Milan |  |  |  |  |  |  |
| C | $\emptyset$ sterbro | København K | CS | 12 | 0 | 5.95 | 0 | 0 | M | Portobello | Derganino | CS | 11 | 0 | 5.95 | 0 | 0 |
| C | $\emptyset$ sterbro | København K | PT | 0 | 12 | 3.27 | 0 | 5 | M | Portobello | Derganino | PT | 0 | 12 | 11.31 | 0 | 6 |
| C | Østerbro | København K | B | 0 | 0 | 0 | 13.50 | 0 | M | Portobello | Derganino | B | 0 | 0 | 0 | 13.13 | 0 |
| C | Østerbro | Nørrebro | CS | 11 | 0 | 5.95 | 0 | 0 | M | Portobello | China Town | CS | 9 | 0 | 5.95 | 0 | 0 |
| C | $\emptyset$ sterbro | Nørrebro | PT | 0 | 10 | 6.01 | 0 | 6 | M | Portobello | China Town | PT | 0 | 6 | 14.29 | 0 | 3 |
| C | $\emptyset$ sterbro | Nørrebro | B | 0 | 0 | 0 | 10.50 | 0 | M | Portobello | Sempione | CS | 9 | 0 | 5.95 | 0 | 0 |
| C | $\emptyset$ sterbro | Fredriksberg C | CS | 16 | 0 | 5.95 | 0 | 0 | M | Portobello | Sempione | PT | 0 | 12 | 6.90 | 0 | 9 |
| C | Østerbro | Fredriksberg C | PT | 0 | 20 | 4.00 | 0 | 13 | M | Portobello | Sempione | B | 0 | 0 | 0 | 10.88 | 0 |
| C | Østerbro | Fredriksberg C | B | 0 | 0 | 0 | 16.50 | 0 | M | Portobello | Washinghton | CS | 11 | 0 | 5.95 | 0 | 0 |
| C | $\emptyset$ sterbro | Frederiksberg | CS | 18 | 0 | 5.95 | 0 | 0 | M | Portobello | Washinghton | PT | 0 | 5 | 23.81 | 0 | 5 |
| C | $\varnothing$ sterbro | Frederiksberg | PT | 0 | 24 | 3.05 | 0 | 9 | M | Portobello | Washinghton | B | 0 | 0 | 0 | 13.50 | 0 |
| C | Østerbro | Frederiksberg | B | 0 | 0 | 0 | 21 | 0 | M | Portobello | Carrobbio | CS | 18 | 0 | 5.95 | 0 | 0 |
| C | $\varnothing$ sterbro | Vesterbro | CS | 21 | 0 | 5.95 | 0 | 0 | M | Portobello | Carrobbio | PT | 0 | 13 | 13.69 | 0 | 6 |
| C | Østerbro | Vesterbro | PT | 0 | 30 | 5.15 | 0 | 4 | M | Portobello | Carrobbio | B | 0 | 0 | 0 | 21.75 | 0 |
| C | $\emptyset$ sterbro | Vesterbro | B | 0 | 0 | 0 | 23.63 | 0 | M | Portobello | Ticinese | CS | 21 | 0 | 5.95 | 0 | 0 |
| C | Østerbro | $\emptyset$ restad | CS | 22 | 0 | 5.95 | 0 | 0 | M | Portobello | Ticinese | PT | 0 | 19 | 11.31 | 0 | 10 |
| C | $\emptyset$ sterbro | $\emptyset$ restad | PT | 0 | 23 | 13.31 | 0 | 9 | M | Portobello | Ticinese | B | 0 | 0 | 0 | 17.63 | 0 |
| C | $\varnothing$ sterbro | $\varnothing$ restad | B | 0 | 0 | 0 | 33 | 0 | M | Portobello | Guastalla | CS | 27 | 0 | 5.95 | 0 | 0 |
| C | Osterbro | Øst Amager | CS | 25 | 0 | 5.95 | 0 | 0 | M | Portobello | Guastalla | PT | 0 | 15 | 19.64 | 0 | 8 |
| C | Østerbro | ¢st Amager | PT | 0 | 21 | 7.73 | 0 | 9 | M | Portobello | Guastalla | B | 0 | 0 | 0 | 27.38 | 0 |
| C | Østerbro | Øst Amager | B | 0 | 0 | 0 | 29.25 | 0 | M | Portobello | QDM | CS | 21 | 0 | 5.95 | 0 | 0 |
| C | København K | Nørrebro | CS | 11 | 0 | 5.95 | 0 | 0 | M | Portobello | QDM | PT | 0 | 13 | 17.26 | 0 | 5 |



| København K | Nørrebro | T |
| :---: | :---: | :---: |
| København K | Nørrebro | B |
| København K | Fredriksberg C | CS |
| København K | Fredriksberg C | PT |
| København K | Fredriksberg C | B |
| København K | Frederiksberg | CS |
| København K | Frederiksberg | PT |
| København K | Frederiksberg | B |
| København K | Vesterbro | CS |
| København K | Vesterbro | PT |
| København K | Vesterbro | B |
| København K | Ørestad | CS |
| København K | $\emptyset$ restad | PT |
| København K | $\emptyset$ restad | B |
| København K | $\varnothing$ st Amager | CS |
| København K | $\varnothing$ st Amager | PT |
| København K | Øst Amager | B |
| Nørrebro | Fredriksberg C | CS |
| Nørrebro | Fredriksberg C | PT |
| Nørrebro | Fredriksberg C | B |
| Nørrebro | Frederiksberg | CS |
| Nørrebro | Frederiksberg | PT |
| Nørrebro | Frederiksberg | B |
| Nørrebro | Vesterbro | CS |
| Nørrebro | Vesterbro | PT |
| Nørrebro | Vesterbro | B |
| Nørrebro | $\emptyset$ restad | CS |
| Nørrebro | $\emptyset$ restad | PT |
| Nørrebro | $\emptyset$ restad | B |
| Nørrebro | $\emptyset$ st Amager | CS |
| Nørrebro | Øst Amager | PT |
| Nørrebro | $\emptyset$ st Amager | B |
| Fredriksberg C | Frederiksberg | CS |
| Fredriksberg C | Frederiksberg | PT |
| Fredriksberg C | Frederiksberg | B |
| Fredriksberg C | Vesterbro | CS |
| Fredriksberg C | Vesterbro | PT |
| Fredriksberg C | Vesterbro | B |
| Fredriksberg C | $\emptyset$ restad | CS |
| Fredriksberg C | $\emptyset$ restad | PT |
| Fredriksberg C | $\emptyset$ restad | B |
| Fredriksberg C | $\varnothing$ st Amager | CS |
| Fredriksberg C | $\varnothing_{\text {st }}$ Amager | PT |
| Fredriksberg C | $\emptyset$ st Amager | B |
| Frederiksberg | Vesterbro | CS |
| Frederiksberg | Vesterbro | PT |
| Frederiksberg | Vesterbro | B |
| Frederiksberg | $\emptyset$ restad | CS |
| Frederiksberg | $\emptyset$ restad | PT |
| Frederiksberg | $\emptyset$ restad | B |
| Frederiksberg | $\emptyset$ st Amager | CS |
| Frederiksberg | $\emptyset$ st Amager | PT |
| Frederiksberg | $\emptyset$ st Amager | B |
| Vesterbro | $\emptyset$ restad | CS |
| Vesterbro | $\emptyset$ restad | PT |
| Vesterbro | $\emptyset$ restad | B |
| Vesterbro | Øst Amager | CS |
| Vesterbro | $\emptyset$ st Amager | PT |
| Vesterbro | Øst Amager | B |
| Ørestad | Øst Amager | CS |
| Ørestad | Øst Amager | PT |
| $\emptyset$ restad | $\emptyset$ st Amager | B |
| Washinghton | Carrobbio | CS |
| Washinghton | Carrobbio | PT |
| Washinghton | Carrobbio | B |
| Washinghton | Ticinese | CS |
| Washinghton | Ticinese | PT |
| Washinghton | Ticinese | B |
| Washinghton | Guastalla | CS |
| Washinghton | Guastalla | PT |
| Washinghton | Guastalla | B |
| Washinghton | QDM | CS |
| Washinghton | QDM | PT |
| Washinghton | QDM | B |
| Washinghton | Central Station | CS |
| Washinghton | Central Station | PT |
| Washinghton | Central Station | B |
| Ticinese | Guastalla | CS |
| Ticinese | Guastalla | PT |
| Ticinese | Guastalla | B |
| Ticinese | QDM | CS |
| Ticinese | QDM | PT |
| Ticinese | QDM | B |
| Ticinese | Central Station | CS |
| Ticinese | Central Station | PT |
| Ticinese | Central Station | B |










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[^1]:    ${ }^{1}$ Source: www.car2go.com accessed on January 6th 2018.

[^2]:    ${ }^{2}$ Assuming the usage of a widely available transportation card named rejsekort.

